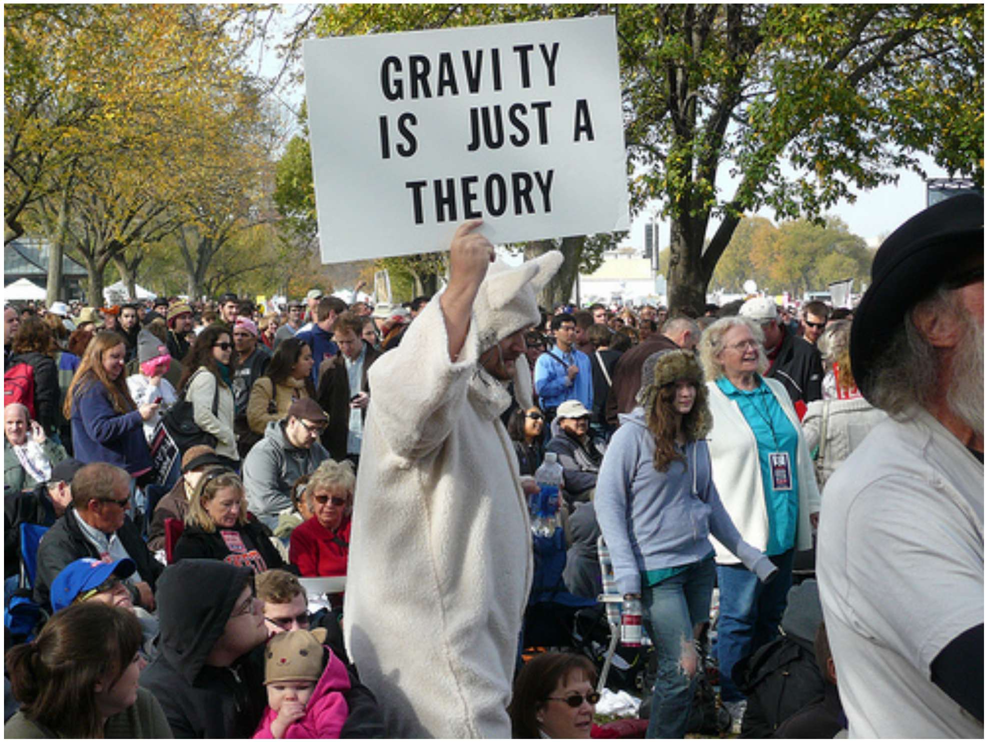
Modified Gravity
A brief introduction



- Why bother?
- Modifying Gravity
- Current knowledge
- Screening
- Parameterisations

Donnacha Kirk, UCL

Thu 7th March 2013
LCDM at QMUL

Why Modify Gravity?...

No evidence GR is broken so why bother?

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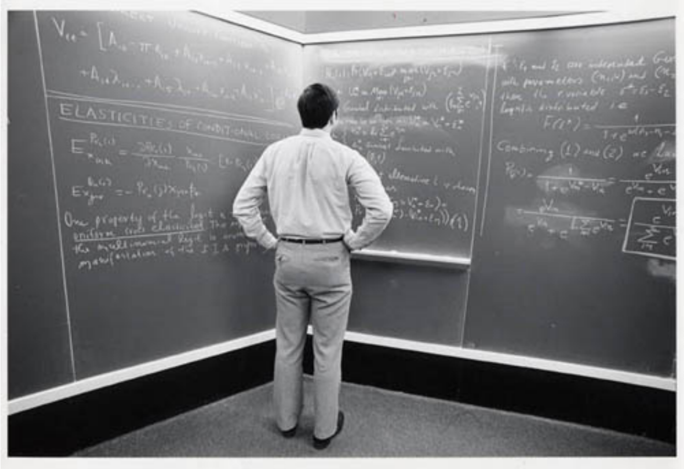
- Because we can…

Whitehead (1922), Cartan (1922, 1923), Fierz & Pauli (1939), Birkhov (1943), Milne (1948), Thiry (1948), Papapetrou (1954a, 1954b), Littlewood (1953), Jordan (1955), Bergman (1956), Belinfante & Swihart (1957), Yilmaz (1958, 1973), Brans & Dicke (1961), Whitrow & Morduch (1960, 1965), Kustaanheimo (1966), Kustaanheimo & Nuotio (1967), Deser & Laurent (1968), Page & Tupper (1968), Bergmann (1968), Bollini-Giambiagi-Tiomno (1970), Nordtveldt (1970), Wagoner (1970), Rosen (1971, 1975, 1975), Wei-Tou Ni (1972, 1973), Will & Nordtveldt (1972), Hellings & Nordtveldt (1973), Lightman & Lee (1973), Lee, Lightman & Ni (1974), Bekenstein (1977), Barker (1978), Rastall (1979)

Theorists begin to modify Einstein's equations as soon as they're published.



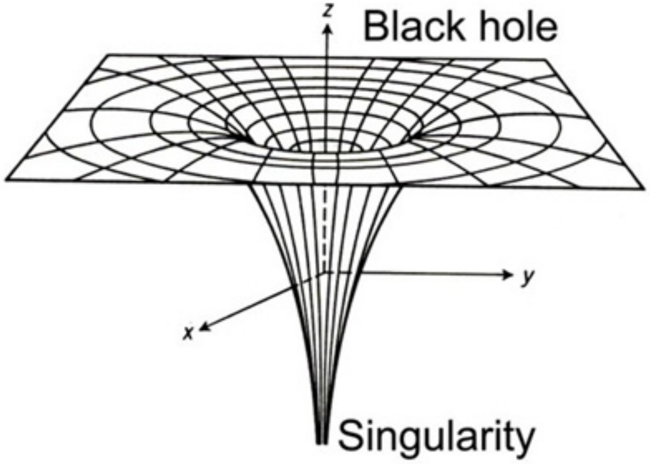
Why Modify Gravity?…

No evidence GR is broken so why bother?

- Because we can…

- Because there are regimes where GR fails

Collapsing black holes, ultra-high energies in the early Universe
→ search for quantum gravity.



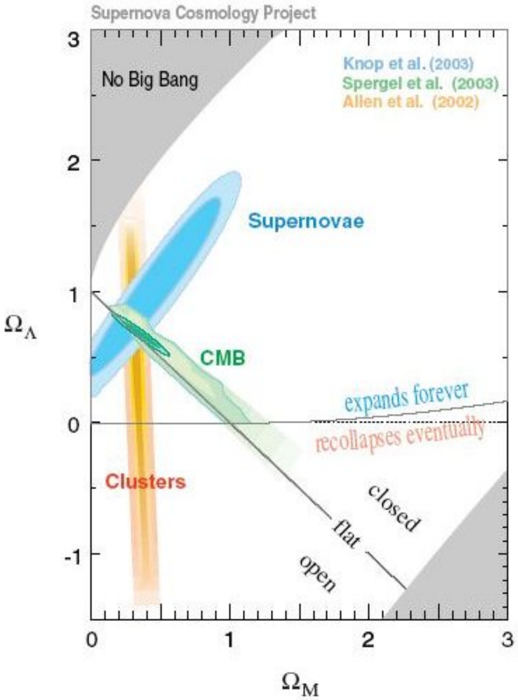
Why Modify Gravity?…

No evidence GR is broken so why bother?

- Because we can…
- Because there are regimes where GR fails
- Because Cosmologists have an embarrassing problem.

MG for Cosmic Acceleration

- We have a concordance cosmology which explains existing observations well….
- … but it requires that ~80% of all matter be 'dark'…
- …. And that ~75% of the entire Universe be exotic 'dark energy'



- Occam's razor favors a cosmological constant. " Dark energy is the cosmological constant until proven otherwise, for the same reason that the moon is not made of cheese until proven otherwise." (Bousso 2007)

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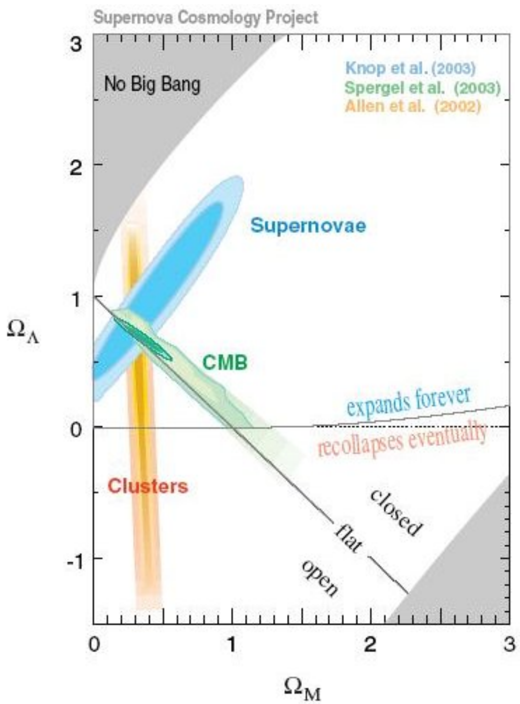

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Λ is boring!

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

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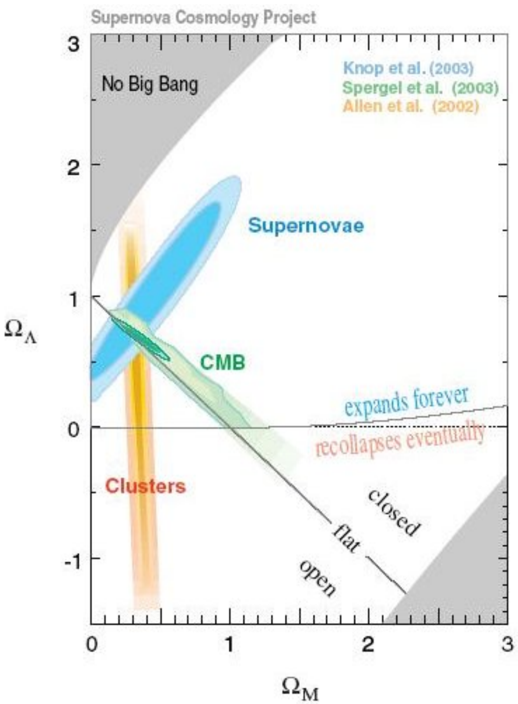

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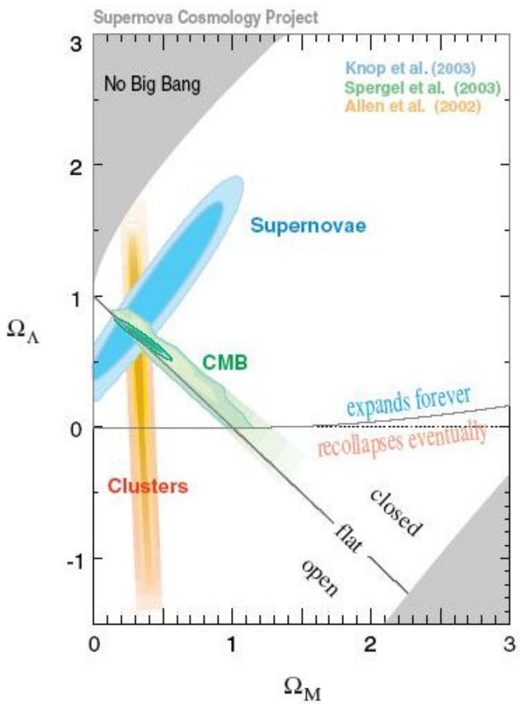

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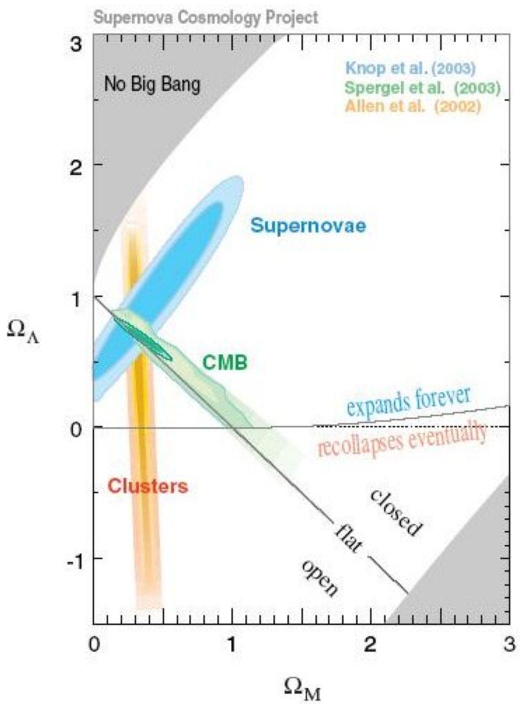

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$$
\begin{aligned}
G_{\mu\nu} &= 8\pi G T_{\mu\nu} \\
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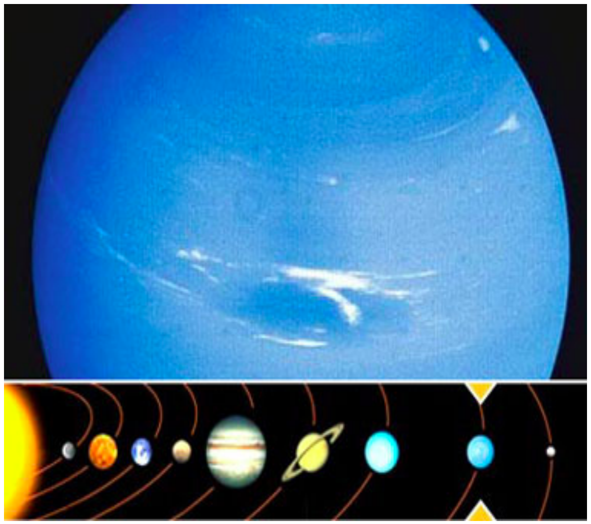
G_{\mu\nu} + G^{dark}_{\mu\nu} &= 8\pi G T_{\mu\nu}
\end{aligned}
$$

- In 2006 **512** papers put on arxiv with Dark Energy in their abstract, **129** with Modified Gravity.
- In 2012 **591** papers put on arxiv with Dark Energy in their abstract, **287** with Modified Gravity.

MG for Cosmic Acceleration

- There are historical precedents for this DE v. MG dilemma.



Errors in calculation of Uranus' orbit
→ Le Verrier's prediction & discovery of **Neptune**,
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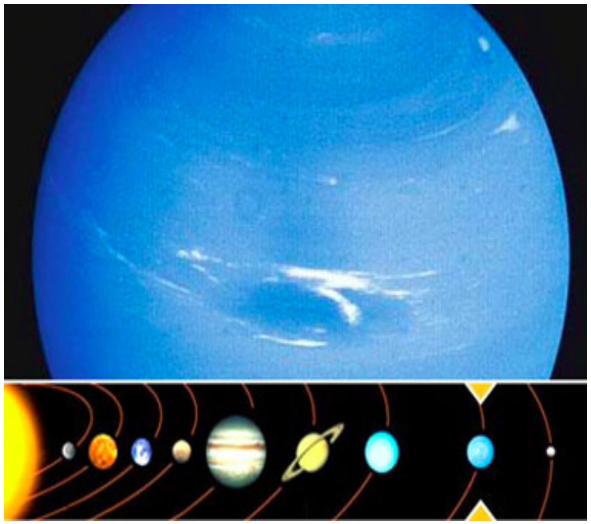

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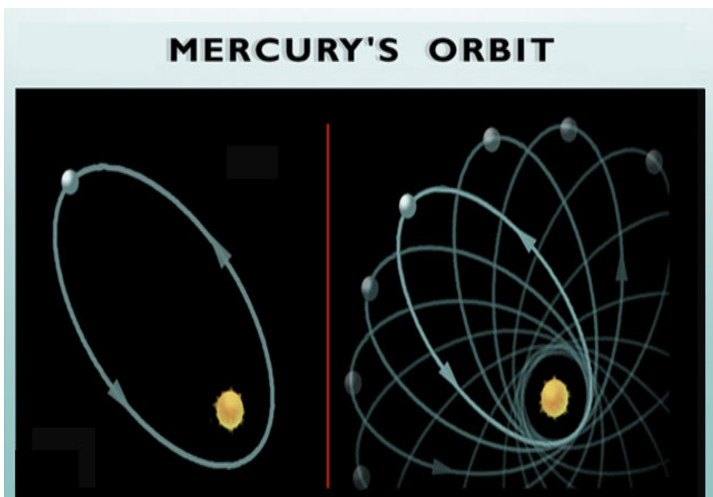

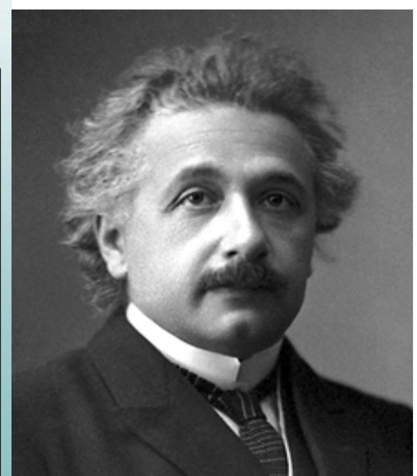

Errors in calculation of precession of **Mercury's** orbit
→ predictions of Vulcan, a new planet.
Searches were fruitless and it required Einstein's GR, a **modified gravity theory**, to explain the observations.

MG Theories

There are very many ways to modify the Einstein equations.
Two simple examples:

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f(R) Simplest generalisation of the Einstein-Hilbert action

$$\int \mathrm{d}^4x\sqrt{-g}\frac{1}{16\pi G_N}R \rightarrow \int \mathrm{d}^4x\sqrt{-g}\frac{1}{16\pi G_N}f(R)$$

Can produce late-time acceleration. Sub-class of chameleon/scalar-tensor

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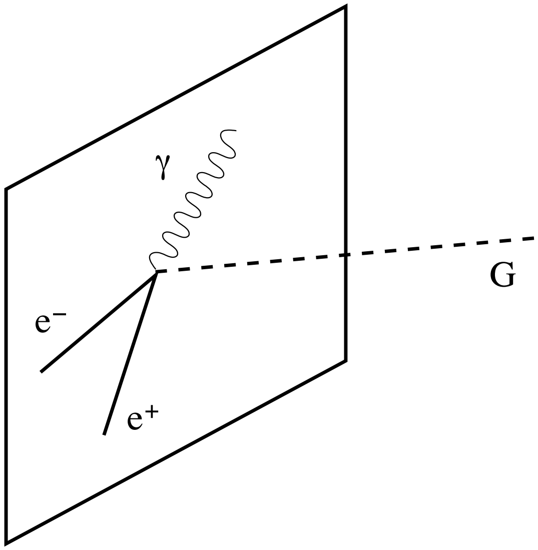
DGP Embed 3+1 spacetime in 5D bulk.

$$S = \frac{1}{32\pi G r_c}\int d^5x\sqrt{-g_{(5)}}R_{(5)} + \frac{1}{16\pi G}\int d^4x\sqrt{-g}R + \int d^4x\sqrt{-g}L_m$$



Gravity 'leaks' off brane, weakened at large scales

MG Theories

- Covariant Galileons
- Horndeski
- K-inflation
- Randall-Sundrum
- MOND
- Degravitation
- Bimetric theories
- f(G)
- General Scalar-Tensor
- Einsten-Aether
- Kinetic Gravity Braiding
- Massive Gravity
- Horava-Lifschitz
- Brans-Dicke
- Cascading gravity
- TeVeS
- Fab Four
- and more...
- EBI gravity

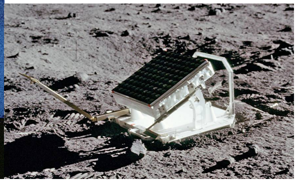
Common problems in MG theories

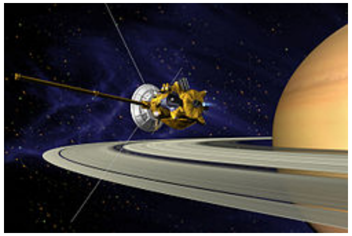
- **Ghosts**- kinetic term has wrong sign. Field speeds up as it climbs potential.
- **Tachyons**- potential not bounded from below, $m^2 < 0$.
- **Superluminal motions** & causality.
- Breaches of **Lorentz invariance**.
- Significant **fine-tuning** to be cosmologically useful.
- Violation of **solar-system constraints** → screening mechanisms.

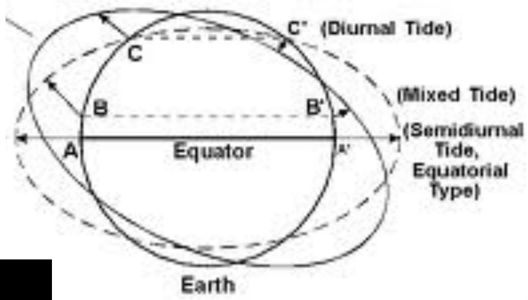
What do we know about Gravity?

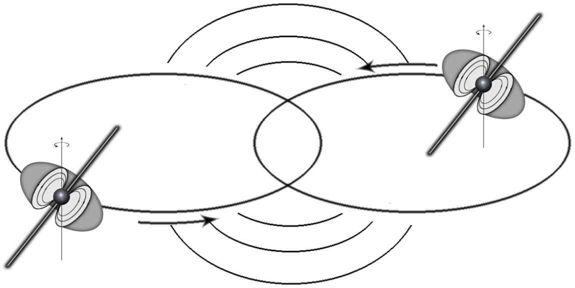
Stringent tests exist on a range of ‘local’ scales.

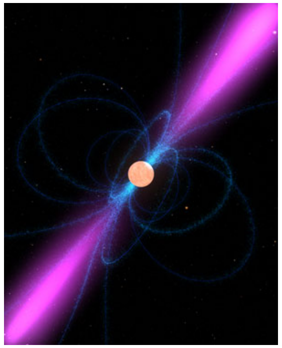
- Eöt-Wash- test WEP in the lab.
- Anomalous Tides.
- Laser Lunar Ranging- Nordtvedt effect.
- Time delays- Cassini
- Gravitational bending of light.
- Precession of perihelion of Mercury.
- Pulsar Timing.
- Binary Pulsars.

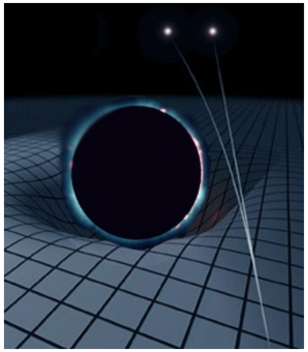












What do we know about Gravity?

- Parameterised Post-Newtonian (PPN) formalism studies perturbations in the slow motion, weak field limit.
- Uses 10 parameters to characterise the coefficients of the metric potentials.

PPN parameters

parameter	What it measures relative to GR
γ	How much space-curvature produced by unit rest mass ?
β	How much "nonlinearity" in the superposition law for gravity ?
ξ	Preferred-location effects ?
$\alpha_1,\alpha_2,\alpha_3$	Preferred-frame effects ?
$\alpha_3,\zeta_1,\zeta_2,\zeta_3,\zeta_4$	Violation of conservation of total momentum ?

metric

$$g_{00}=-1+2U-2\beta U^2-2\xi\Phi_W+(2\gamma+2+\alpha_3+\zeta_1-2\xi)\Phi_1+2(3\gamma-2\beta+1+\zeta_2+\xi)\Phi_2+2(1+\zeta_3)\Phi_3$$

$$+2(3\gamma+3\zeta_4-2\xi)\Phi_4-(\zeta_1-2\xi)\mathcal{A}-(\alpha_1-\alpha_2-\alpha_3)w^2U-\alpha_2w^iw^jU_{ij}+(2\alpha_3-\alpha_1)w^iV_i+\mathcal{O}(\epsilon^3),$$

$$g_{0i}=-(4\gamma+3+\alpha_1-\alpha_2+\zeta_1-2\xi)V_i/2-(1+\alpha_2-\zeta_1+2\xi)W_i/2-(\alpha_1-2\alpha_2)w^iU/2-\alpha_2w^jU_{ij}+\mathcal{O}(\epsilon^{5/2}),$$

$$g_{ij}=(1+2\gamma U)\delta_{ij}+\mathcal{O}(\epsilon^2).$$

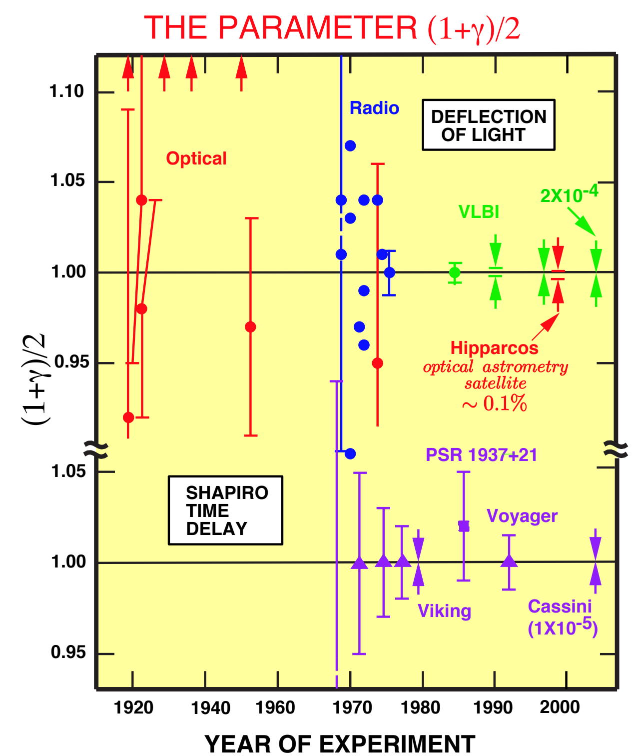
What do we know about Gravity?

Stringent tests exist on a range of ‘local’ scales.

Current limits on the PPN parameters

parameter	Effect	Limit	Remarks
$\gamma - 1$	time delay	2.3×10^{-5}	Cassini tracking
	light deflection	4×10^{-4}	VLBI
$\beta - 1$	perihelion shift	3×10^{-3}	$J_2 = 10^{-7}$ from helioseismology
	Nordtvedt effect	2.3×10^{-4}	$\eta_N = 4\beta - \gamma - 3$ assumed
ξ	Earth tides	10^{-3}	gravimeter data
α_1	orbital polarization	10^{-4}	Lunar laser ranging
		2×10^{-4}	PSR J2317+1439
α_2	spin precession	4×10^{-7}	solar alignment with ecliptic
α_3	pulsar acceleration	4×10^{-20}	pulsar $\dot{P}$ statistics
η_N	Nordtvedt effect	9×10^{-4}	Lunar laser ranging
ζ_1	—	2×10^{-2}	conbined PPN bounds
ζ_2	binary acceleration	4×10^{-5}	$\ddot{P}_{\rm p}$ for PSR 1913+16
ζ_3	Newton's 3rd law	10^{-8}	lunar acceleration
ζ_4	—	6×10^{-3}	$6\zeta_4 = 3\alpha_3 + 2\zeta_1 - 3\zeta_3$ assumed

Sotani 2009



No observed deviations from GR.

UCL
Donnacha Kirk
LCDM@QMUL, 07 March 2013

What do we know about Gravity?

Gravity at larger, cosmologically interesting scales is comparatively poorly constrained.

We do have a range of cosmological probes available at a variety of scales which are differently sensitive to metric perturbations.

Combined probes break degeneracies.

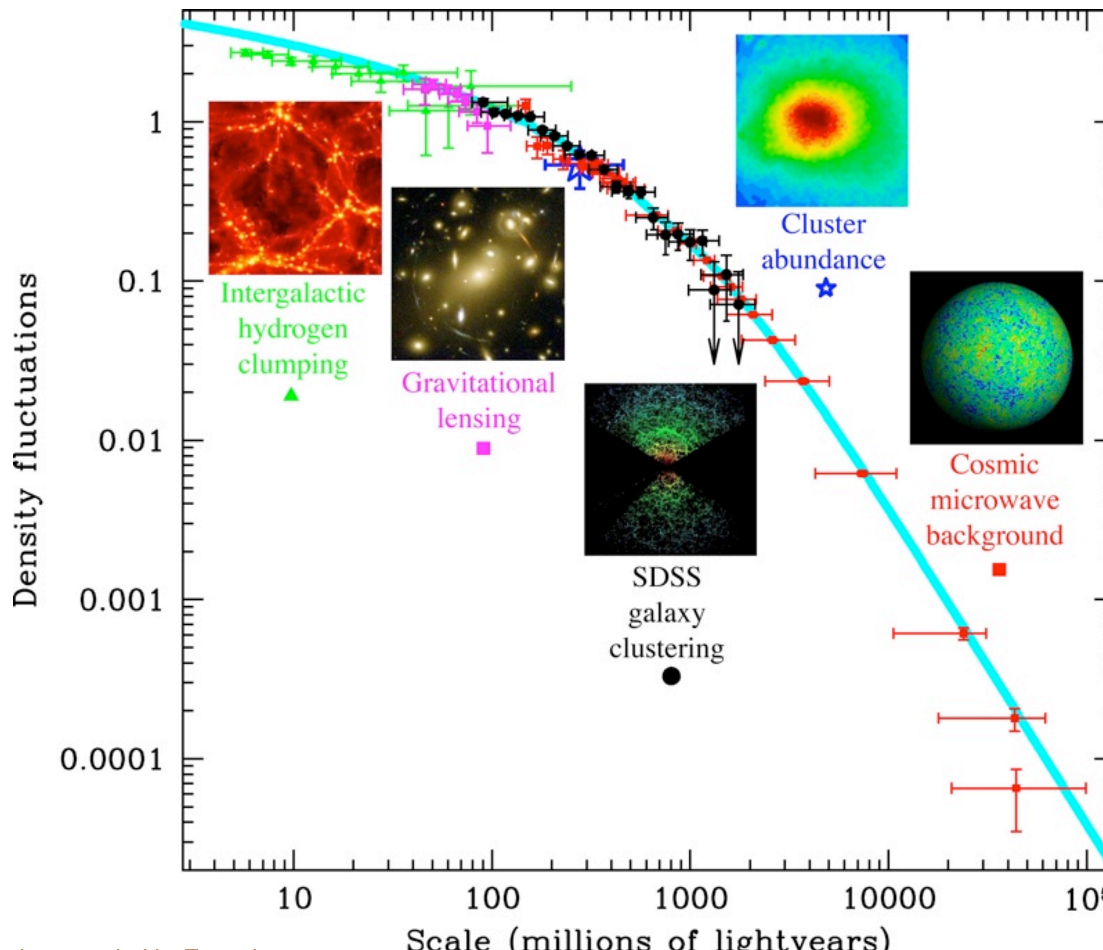


Image credit: Max Tegmark

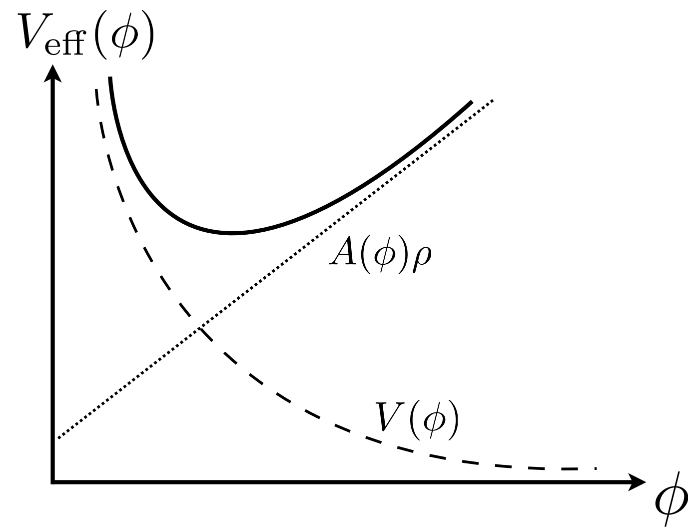
Screening Mechanisms

- Non-linear mechanism to recover GR locally.

Chameleon- modifiy Einstein-Hilbert action to include a scalar potential V(phi) and a more general coupling, A(phi), to matter fields.

$$S_{\mathrm{cham}} = \int \mathrm{d}^4x\sqrt{-g}\left(\frac{M_{\mathrm{Pl}}^2}{2}R - \frac{1}{2}(\partial\phi)^2 - V(\phi)\right) + S_{\mathrm{matter}}\left[g_{\mu\nu}A^2(\phi)\right]$$

$$V_{\mathrm{eff}}(\phi) = V(\phi) + A(\phi)\rho$$



- Local density: 1 g/cm^3
- Cosmic density: 1x10^{-30}g/cm^3

Screening Mechanisms

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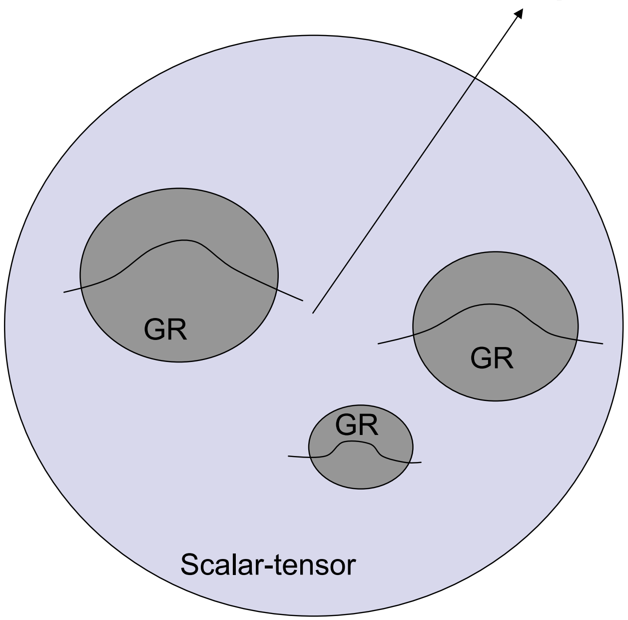
Symmetron- scalar field, small mass everywhere. VEV depends on local mass density. VEV large in low mass regions and vice versa. Coupling of scalar to matter is proportional to VEV

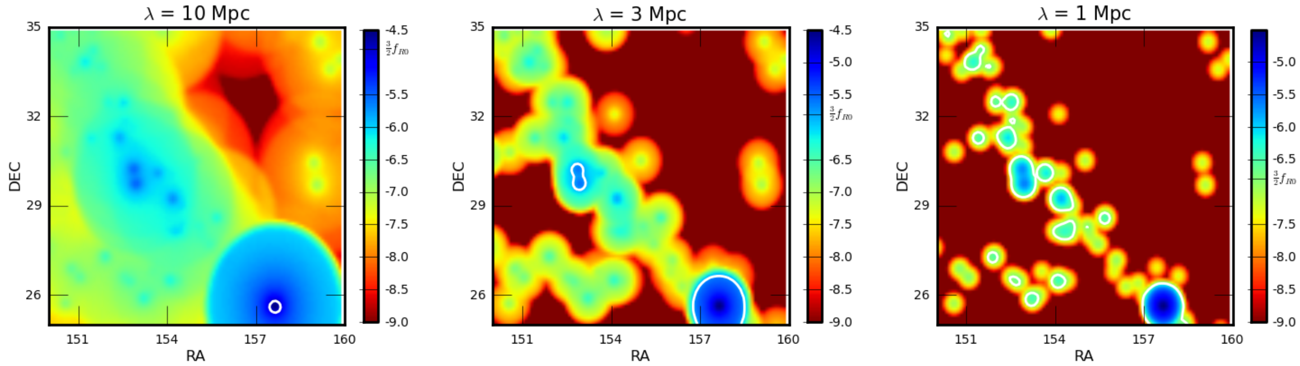
Vainshtein- non-linearities in the longitudinal mode of the graviton dominate in the presence of astrophysical sources → decouple from matter, applied to theories of massive/resonance gravity e.g. DGP



Screening Mechanisms

Possible to experimentally test screening mechanisms astrophysically.

Make a 'screening map'.



Cabre et al. 2012

Look for unscreened objects e.g. isolated dwarf galaxies.

Jain & Vanderplas 2011

Parameterisations

- Don't attempt a consistent theory- look for generic deviations from GR.
- Labour-saving for both theoreticians & observers.
- Assume gravity theory reproduces the phenomenology of an effective w

→ Look for signatures in the growth of structure which differentiate MG/DE.

1 parameter formalism.

γ parameterises the growth of structure.

$$f \equiv \frac{d\ ln\ D}{d\ ln\ a} = \frac{\dot{\delta}}{\delta} \equiv \Omega_m^{\gamma}(a)$$

γ = 0.55 in GR
γ = 0.68 in DGP etc.

Parameterisations

2 parameter formalism.

- Cosmological perturbation theory in the weak-field limit where we can linearise the Einstein equations. 4 variables.

$$\delta, \theta_v, \Psi, \Phi \qquad \mathrm{d}s^2 = -(1+2\Psi)\mathrm{d}t^2 + (1-2\Phi)a^2(t)\mathrm{d}\mathbf{x}^2$$

- Need 4 equations: 2 come from conservation of energy-momentum, 2 from a theory of gravity:

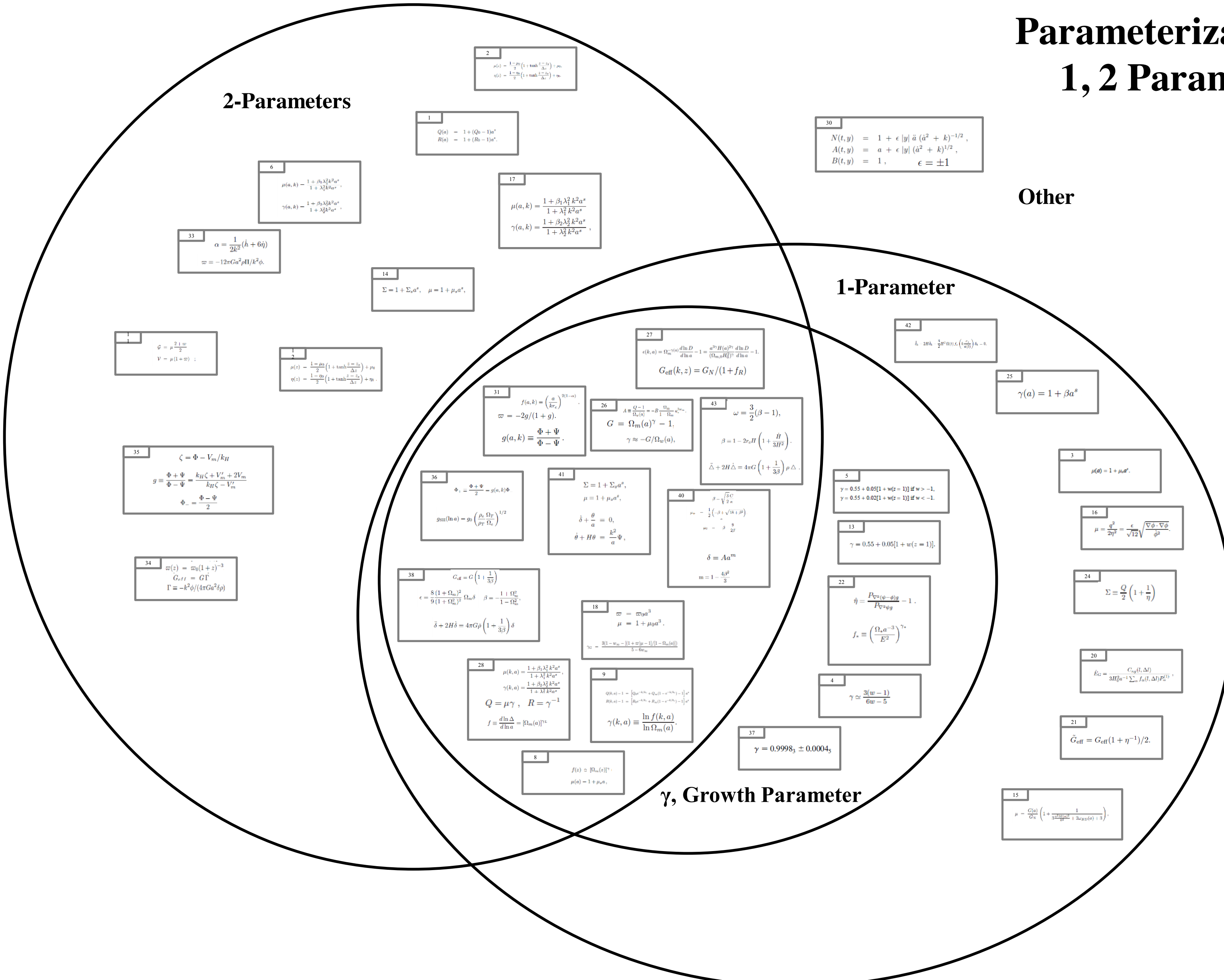
$$k^2\Psi = -4\pi\mu(k,z)Ga^2\rho\Delta \qquad \Phi = \gamma(k,z)\Psi$$

In GR: $\mu(k,z) = 1$

$\gamma(k,z) = 1$

No agreement on form of these parameters, let alone notation.

Parameterizations
1, 2 Parameters



Owen et al. 2013 (in prep.)

Parameterisations

How useful is the 2 parameter formalism?

- Motivated, at least in the quasi-static approx.
- Convenient: 2 parameters can easily be put into a modified Einstein-Boltzmann solver.

but…

- Free functional form- hard to constrain arbitrary function of scale/redshift.
- Only valid at sub-horizon scales, linear regime.
- Doesn't cover the full theory space.
- Even a smoking gun detection won't necessary lead to a particular theory.
- Is there a smoking gun without a particular theory?

e.g. could clustering dark energy mimic any signature?

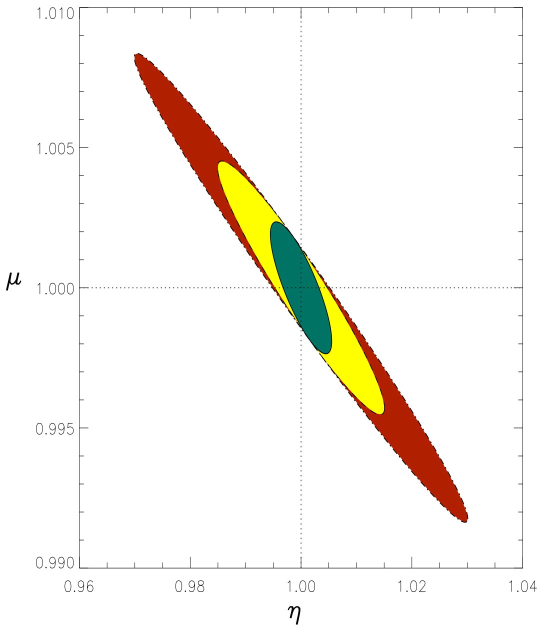


FIG. 5: Fisher constraints on η and μ from cross-correlated CMB and weak lensing measurements (Stage I) are shown by the red area (dot-dashed contour). The improvement obtained by adding Stage I cluster counts is seen in the green area (solid contour). The outer ellipse corresponds to the inner ellipse of Fig. 4. Adding an uncertainty in the mass assignment for the clusters of $\sigma_{\mathcal{M}} = 0.25$ decreases the impact of adding cluster data as shown by the yellow (dashed) ellipse.

Thomas & Contaldi 2011

Parameterisations

Parameterised Post-Friedmann (PPF) formalism

- Attempt to do for cosmological tests what PPN did for solar-system tests.
- Relevant all the way to horizon scales.
- Requires a large number of free Functions- ambitious.
- Allows any/many(?) MG theories to be expressed in the same language.

$$A_0 = -2\left(1 - \frac{a\tilde{\Theta}}{\mathcal{H}}\right) - \frac{\dot{\phi}}{\mathcal{H}}\frac{a^2}{k^2}\tilde{\mu} + \frac{2}{\mathcal{H}^2 k^2}\left(\dot{\mathcal{H}} - \mathcal{H}\frac{\ddot{\phi}}{\dot{\phi}}\right)(a^2\tilde{\Upsilon} + 3\mathcal{H}a\tilde{\Theta})$$

$$B_0 = \frac{1}{k\mathcal{H}}\left(\kappa a^2\rho_M - 2(\mathcal{H}^2 - \dot{\mathcal{H}})\frac{\tilde{\Theta}a}{\mathcal{H}}\right)$$

$$C_0 = 2(1 - \tilde{\mathcal{G}}_T) - 2\frac{\dot{\tilde{\mathcal{G}}}_T}{\mathcal{H}}\left(1 + 3\frac{\dot{\mathcal{H}}}{k^2}\right) - 6\frac{\tilde{\mathcal{G}}_T}{k^2}\left(2\dot{\mathcal{H}} + \frac{\ddot{\mathcal{H}}}{\mathcal{H}}\right) - \frac{3\dot{\mathcal{H}}}{k^2\mathcal{H}^2}\kappa a^2\rho_M$$
$$+ \frac{6a\tilde{\Theta}}{\mathcal{H}k^2}\left(4\dot{\mathcal{H}} - 2\frac{\dot{\mathcal{H}}^2}{H} + \frac{\ddot{\mathcal{H}}}{\mathcal{H}}\right) - \frac{12\ddot{\phi}^2}{k^2\dot{\phi}^2}\left(\tilde{\mathcal{G}}_T - \frac{a\tilde{\Theta}}{\mathcal{H}}\right) + \frac{6\dot{\mathcal{H}}a\dot{\tilde{\Theta}}}{\mathcal{H}^2k^2} - \frac{3a^2\tilde{\mathcal{V}}\dot{\phi}}{\mathcal{H}k^2}$$
$$\frac{3}{k^2\dot{\phi}}\left[2\phi^{(3)}\left(\tilde{\mathcal{G}}_T - \frac{a\tilde{\Theta}}{\mathcal{H}}\right) + \ddot{\phi}\left(2\dot{\tilde{\mathcal{G}}}_T - \frac{2a\dot{\tilde{\Theta}}}{\mathcal{H}} + 4\tilde{\mathcal{G}}_T\left(\mathcal{H} + \frac{\dot{\mathcal{H}}}{\mathcal{H}}\right) - 8a\tilde{\Theta} + \frac{1}{\mathcal{H}}\kappa a^2\rho_M\right)\right]$$

$$C_1 = \frac{2k}{\mathcal{H}}(1 - \tilde{\mathcal{G}}_T) + \frac{6}{k\mathcal{H}}(\mathcal{H}^2 - \dot{\mathcal{H}})\left(1 - \frac{\tilde{\Theta}a}{\mathcal{H}}\right)$$

$$D_0 = 1 - \tilde{\mathcal{G}}_T - \frac{\dot{\tilde{\mathcal{G}}}_T}{\mathcal{H}} \qquad D_1 = \frac{k}{\mathcal{H}}(1 - \tilde{\mathcal{G}}_T)$$

$$F_0 = -\frac{2}{k\mathcal{H}}(3\mathcal{H}^2 + a^2\tilde{\Upsilon}) \qquad I_0 = 2\left(1 - \frac{\tilde{\Theta}a}{\mathcal{H}}\right)$$

$$J_0 = \frac{1}{k\mathcal{H}}\left[-2k^2(1 - \tilde{\mathcal{G}}_T) + 3\kappa a^2\rho_M - 6\frac{d}{d\eta}(a\tilde{\Theta}) + 6(\mathcal{H}^2 + \dot{\mathcal{H}}) - 6\frac{\tilde{\Theta}a}{\mathcal{H}}(2\mathcal{H}^2 - \dot{\mathcal{H}})\right]$$

$$J_1 = 6\left(1 - \frac{\tilde{\Theta}a}{\mathcal{H}}\right)$$

$$K_0 = -\frac{k}{\mathcal{H}}(1 - \tilde{\mathcal{G}}_T) \qquad K_1 = 0$$

$$\alpha_0 = M_P\left[\frac{a^2}{k^2}\tilde{\mu} - \frac{2}{\dot{\phi}}\left(\tilde{\Theta}a - \mathcal{H}\tilde{\mathcal{G}}_T\right)\right] \qquad \alpha_1 = \frac{2M_P}{k\dot{\phi}}\left[a^2\tilde{\Upsilon} + 3\mathcal{H}\,a\tilde{\Theta}\right]$$

$$\beta_0 = \frac{M_P}{k\dot{\phi}^2}\left[-2\ddot{\phi}\left(a\tilde{\Theta} - \mathcal{H}\tilde{\mathcal{G}}_T\right) - 2\tilde{\mathcal{G}}_T\dot{\mathcal{H}}\dot{\phi} + 2a\tilde{\Theta}\mathcal{H}\dot{\phi}\right] - M_P\frac{\kappa a^2\rho_M}{k\dot{\phi}} \qquad \beta_1 = \frac{2M_P}{\dot{\phi}}\left[\tilde{\Theta}a - \mathcal{H}\tilde{\mathcal{G}}_T\right]$$

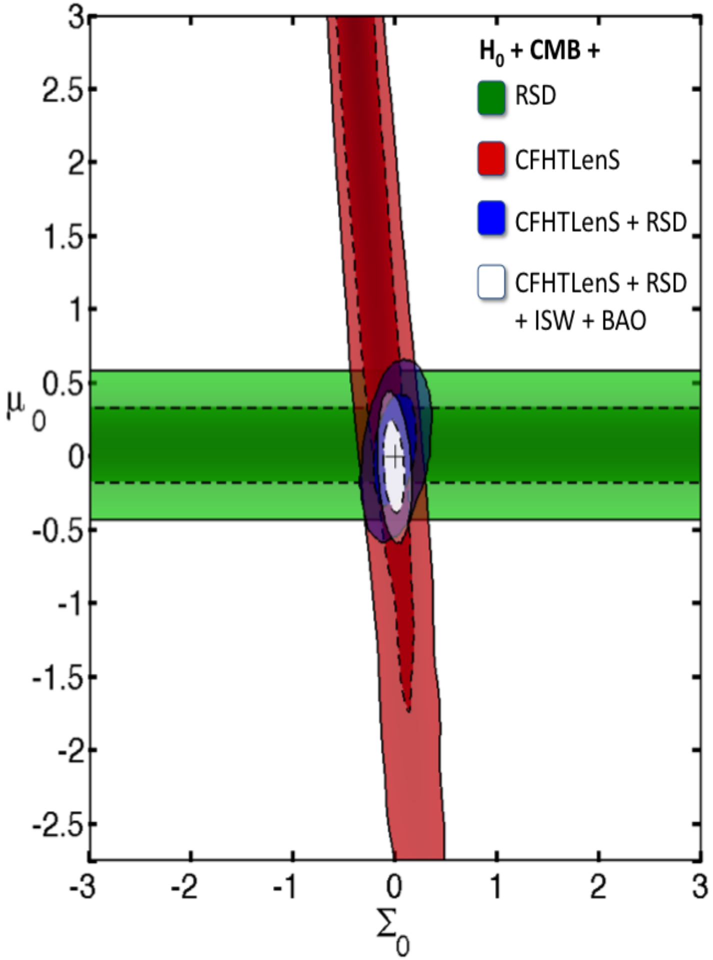
$$\gamma_0 = \frac{2M_P}{\dot{\phi}}\left(\dot{\tilde{\mathcal{G}}}_T + \mathcal{H}\tilde{\mathcal{G}}_T - \mathcal{H}\tilde{\mathcal{F}}_T\right) + 3M_P\frac{a^2}{k^2}\tilde{\mathcal{V}} \qquad \gamma_2 = \frac{6M_P}{\dot{\phi}}\left(\tilde{\Theta}a - \mathcal{H}\tilde{\mathcal{G}}_T\right)$$

$$\gamma_1 = \frac{M_P}{k\dot{\phi}}\left[-6\tilde{\mathcal{G}}_T\left(\dot{\mathcal{H}} + 2\mathcal{H}^2 - 2\mathcal{H}\frac{\ddot{\phi}}{\dot{\phi}}\right) + 6\frac{d}{d\eta}\left(a\tilde{\Theta} - \mathcal{H}\tilde{\mathcal{G}}_T\right) + 6a\tilde{\Theta}\left(3\mathcal{H} - 2\frac{\ddot{\phi}}{\dot{\phi}}\right) - 3\kappa a^2\rho_M\right]$$

$$\epsilon_0 = \frac{M_P}{\dot{\phi}}\left[\dot{\tilde{\mathcal{G}}}_T + \mathcal{H}\tilde{\mathcal{G}}_T - \mathcal{H}\tilde{\mathcal{F}}_T\right] \qquad \epsilon_1 = \epsilon_2 = 0$$

Measurements

Current: CFHTLenS+



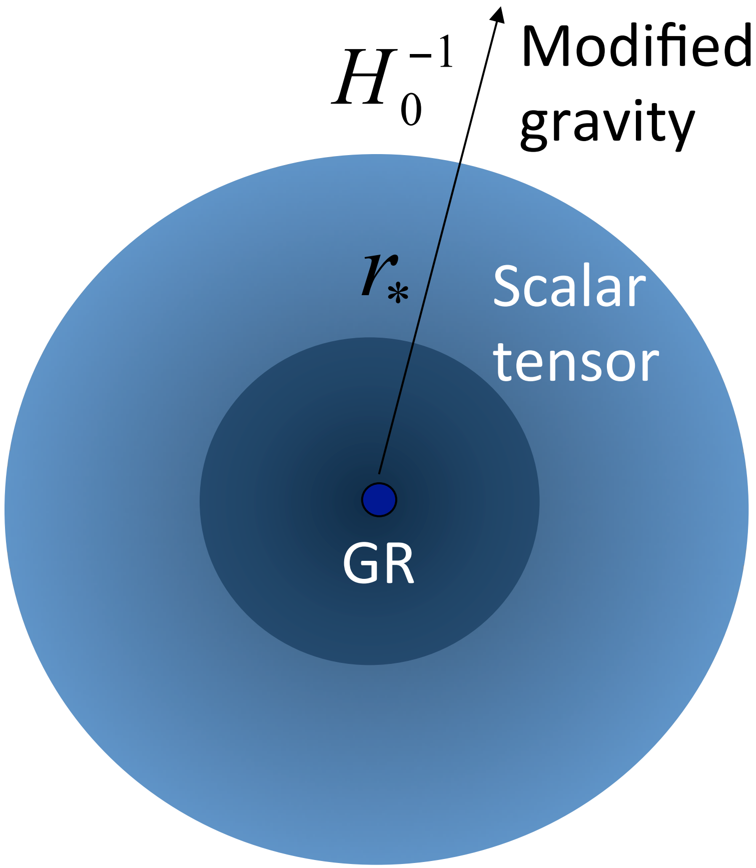
Simpson et al. 2012

- Gravity is being tested at cosmological scales.
- Combination of probes is crucial.
- Constraints will improve by an order of magnitude over next decade.
- Comparison of papers is confusing in the absence of a standard formalism.

Questions for Discussion

- Do we need a reason to test GR?
- Should we be falsifying particular theories or just “testing gravity”?
- Where should we focus our search- where we want our new theory to work or where we expect it to fail?
- Can MG/DE ever be differentiated? Are they really distinct?
- Can a parameterised approach work? Do you have a favourite?
- Where will we be in ten years?

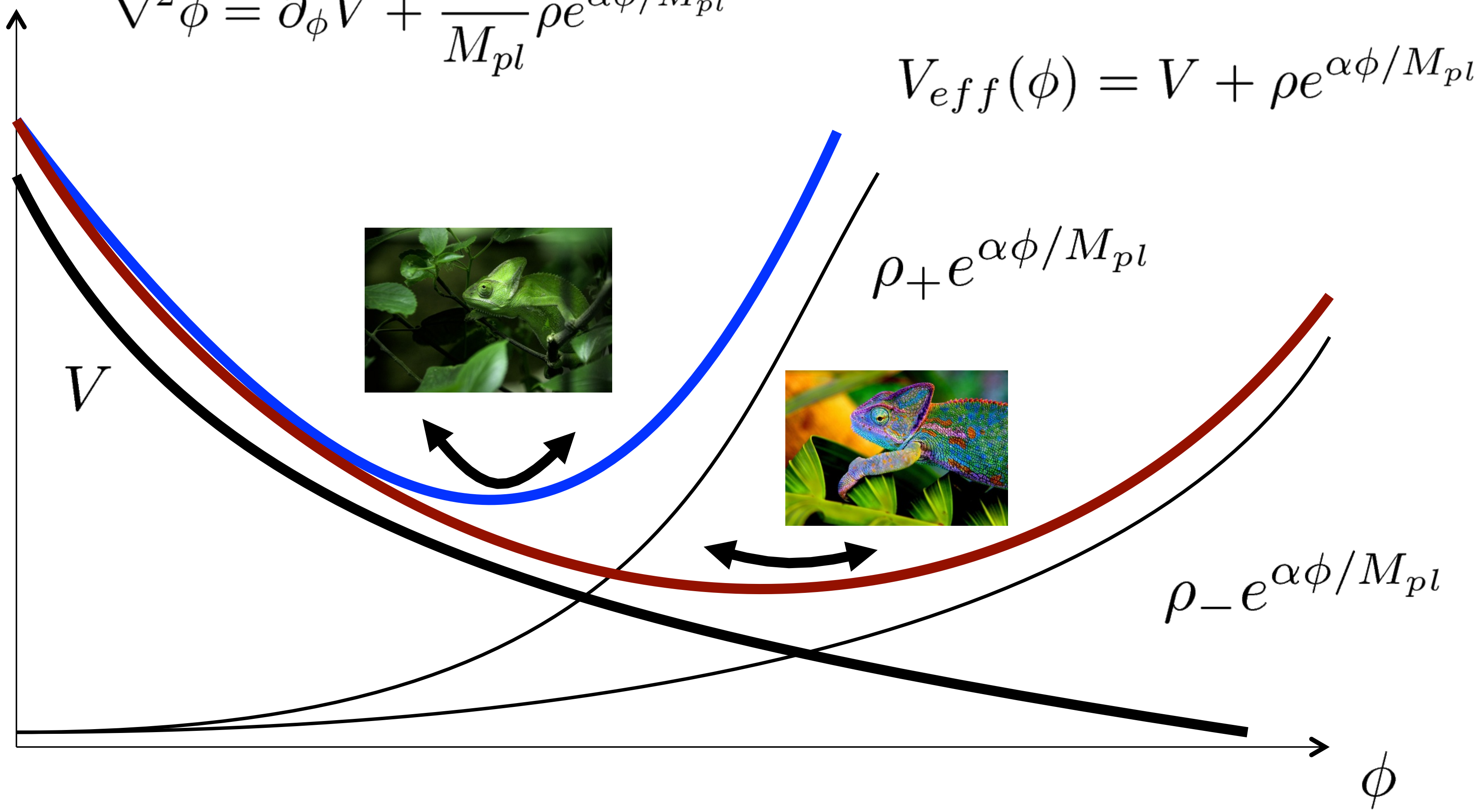
EXTRA SLIDES



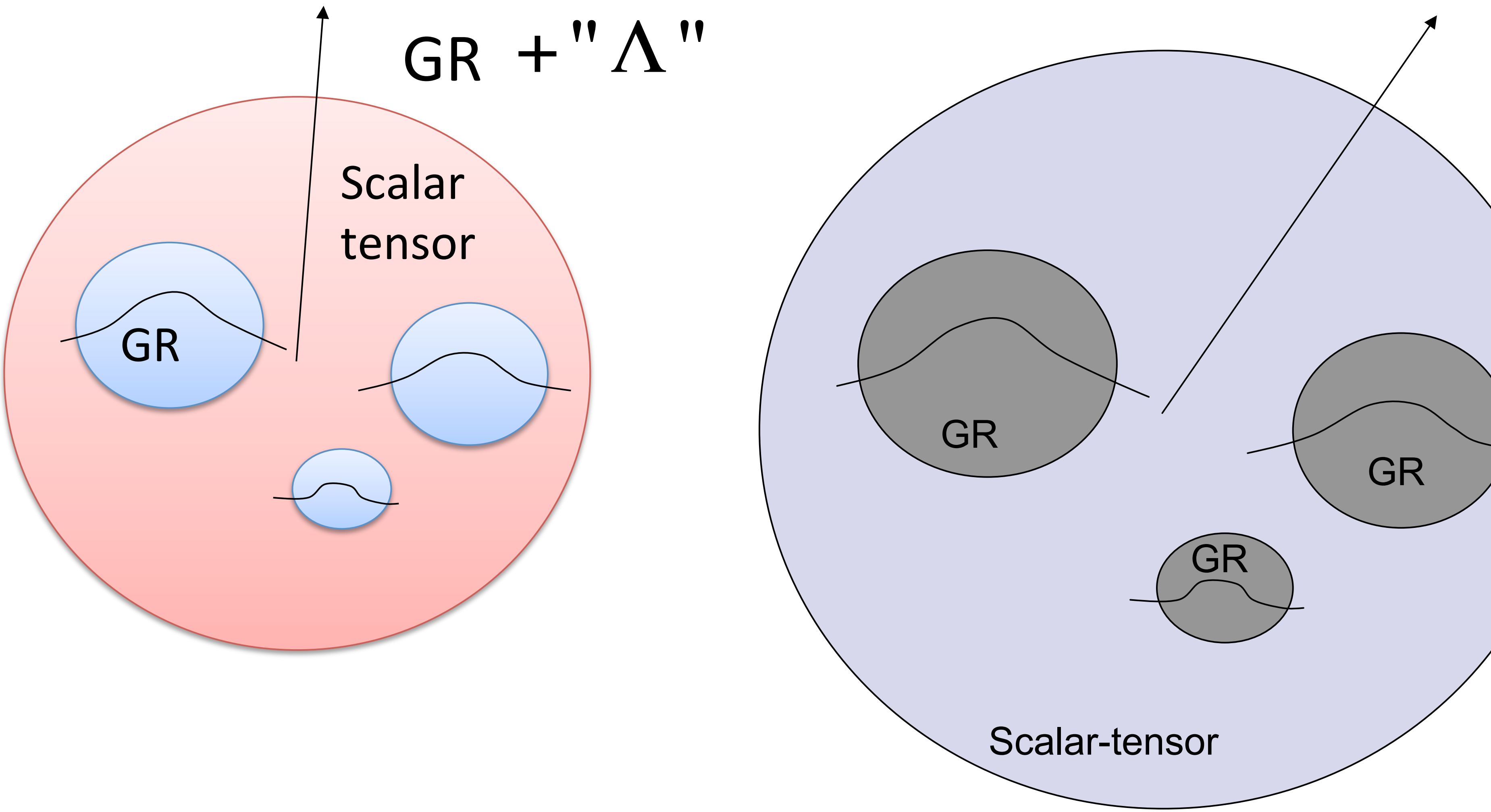
H_0^{-1}
Modified gravity
r_*
Scalar tensor
GR

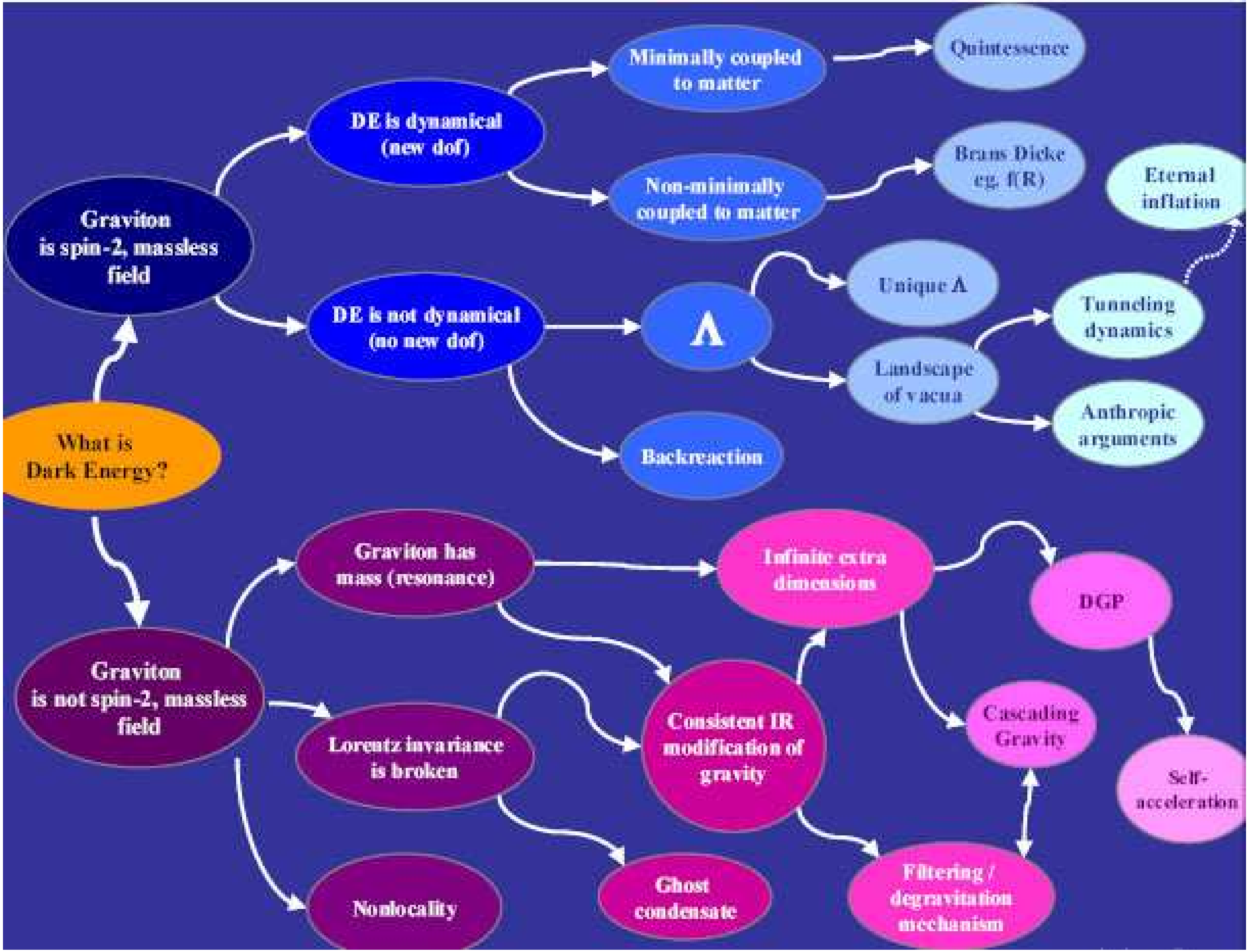
Chameleon mechanism (Khoury & Weltman)

$$\nabla^2\phi = \partial_\phi V + \frac{\alpha}{M_{pl}}\rho e^{\alpha\phi/M_{pl}}$$



Screening Mechanisms




What is Dark Energy?
Graviton is spin-2, massless field
DE is dynamical (new dof)
Minimally coupled to matter
Quintessence
Non-minimally coupled to matter
Brans Dicke eg. f(R)
Eternal inflation
DE is not dynamical (no new dof)
Λ
Unique Λ
Landscape of vacua
Tunneling dynamics
Anthropic arguments
Backreaction
Graviton is not spin-2, massless field
Graviton has mass (resonance)
Infinite extra dimensions
DGP
Self-acceleration
Cascading Gravity
Lorentz invariance is broken
Consistent IR modification of gravity
Filtering / degravitation mechanism
Ghost condensate
Nonlocality

- Four perturbed varaibles: delta, theta, phi, psi

3.2.2. Quasi-static Newtonian Regime

In what follows, we will for the most part make the approximation of non-relativistic motions and restrict ourselves to sub-horizon length scales. One can also self-consistently neglect time derivatives of the metric potentials in comparison to spatial gradients. These approximations will be referred to as the quasi-static, Newtonian regime. Using the linearized fluid equations, the evolution of density (or velocity) perturbations can be described by a single second order differential equation:

$$\ddot{\delta} + 2H\dot{\delta} + \frac{k^2\Psi}{a^2} = 0\,. \tag{85}$$

With $\delta(\vec{k},t) \simeq \delta_{\rm initial}(\vec{k})D(k,t)$, we can substitute for Ψ in terms of δ using the Poisson equation. Here we write the Poisson equation with the sum of potentials on the left-hand side, as this is convenient for describing lensing and the ISW effect. Using the generalized gravitational "constant" $\tilde{G}_{\rm eff}$ we have

$$k^2(\Psi + \Phi) = -8\pi \tilde{G}_{\rm eff}(k,t)\bar{\rho}\delta\,. \tag{86}$$

Using the two equations above, we obtain for the linear growth factor $D(k,t)$:

$$\ddot{D} + 2H\dot{D} - \frac{8\pi\tilde{G}_{\rm eff}}{(1+\Phi/\Psi)}\bar{\rho}\; D = 0\,. \tag{87}$$

From the above equation one sees readily how the combination of $G_{\rm eff}$ and Φ/Ψ alters the linear growth factor. Further, if these parameters have a scale dependence, then even the linear growth factor D becomes scale dependent, a feature not seen in smooth dark energy models.

We will use the power spectra of various observables to describe their scale dependent two point correlations. As an example, the 3-dimensional power spectrum of the density contrast $\delta(k,z)$ is defined as

$$\langle \delta(\vec{k},z)\delta(\vec{k}',z)\rangle = (2\pi)^3\delta_{\rm D}(\vec{k}+k')P_{\delta\delta}(k,z)\,, \tag{88}$$

where we have switched the time variable to the observable redshift z. The power spectra of perturbations in other quantities are defined analogously. We will denote the cross-spectra of two different variables with appropriate subscripts. For example, $P_{\delta\Psi}$ denotes the cross-spectrum of the density perturbation δ and the potential Ψ.

The action in General Relativity

In General Relativity the action that gives Einstein equation is simple:

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\,(R - 2\Lambda) + S_m\,. \tag{2}$$

where $R = g^{\mu\nu}R_{\mu\nu}$ is the Ricci scalar and S_m is a matter action. We take into account the cosmological constant Λ.

The variation of this action with respect to $g^{\mu\nu}$ gives

$$\begin{aligned}\delta S &= \frac{1}{16\pi G}\int d^4x\sqrt{-g}\left[\frac{1}{2}(R-2\Lambda)g^{\mu\nu}\delta g_{\mu\nu} + \delta g^{\mu\nu}R_{\mu\nu} + g^{\mu\nu}\delta R_{\mu\nu}\right] + \delta S_m \\ &= \frac{1}{16\pi G}\int d^4x\sqrt{-g}\left[\left\{\frac{1}{2}g^{\mu\nu}(R-2\Lambda) - R^{\mu\nu}\right\}\delta g_{\mu\nu} + g^{\mu\nu}\delta R_{\mu\nu}\right] + \delta S_m\,,\end{aligned}$$

where we used

$$\delta(\sqrt{-g}) = \frac{1}{2}g^{\mu\nu}\delta g_{\mu\nu}\sqrt{-g}\,, \quad \delta g^{\mu\nu} = -g^{\mu\alpha}g^{\nu\beta}\delta g_{\alpha\beta}\,. \tag{3}$$

Since $\delta R_{\mu\nu} = (\delta\Gamma^{\alpha}_{\mu\nu})_{;\alpha} - (\delta\Gamma^{\alpha}_{\mu\alpha})_{;\nu}$ we have $g^{\mu\nu}\delta R_{\mu\nu} = (g^{\mu\nu}\delta\Gamma^{\alpha}_{\mu\nu} - g^{\mu\alpha}\delta\Gamma^{\nu}_{\mu\nu})_{;\alpha}$.

Thanks to the Gauss's theorem we get $\int d^4x\sqrt{-g}g^{\mu\nu}\delta R^{\mu\nu} = 0$.

The energy momentum tensor $T^{\mu\nu}$ is given by $\delta S_m = \frac{1}{2}\int d^4x\sqrt{-g}T^{\mu\nu}\delta g_{\mu\nu}$.

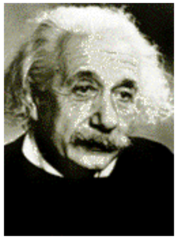
Then the variation δS is

$$\delta S = -\frac{1}{16\pi G}\int d^4x\sqrt{-g}\left[R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R + g^{\mu\nu}\Lambda - 8\pi G T^{\mu\nu}\right]\delta g_{\mu\nu}\,. \qquad (4)$$

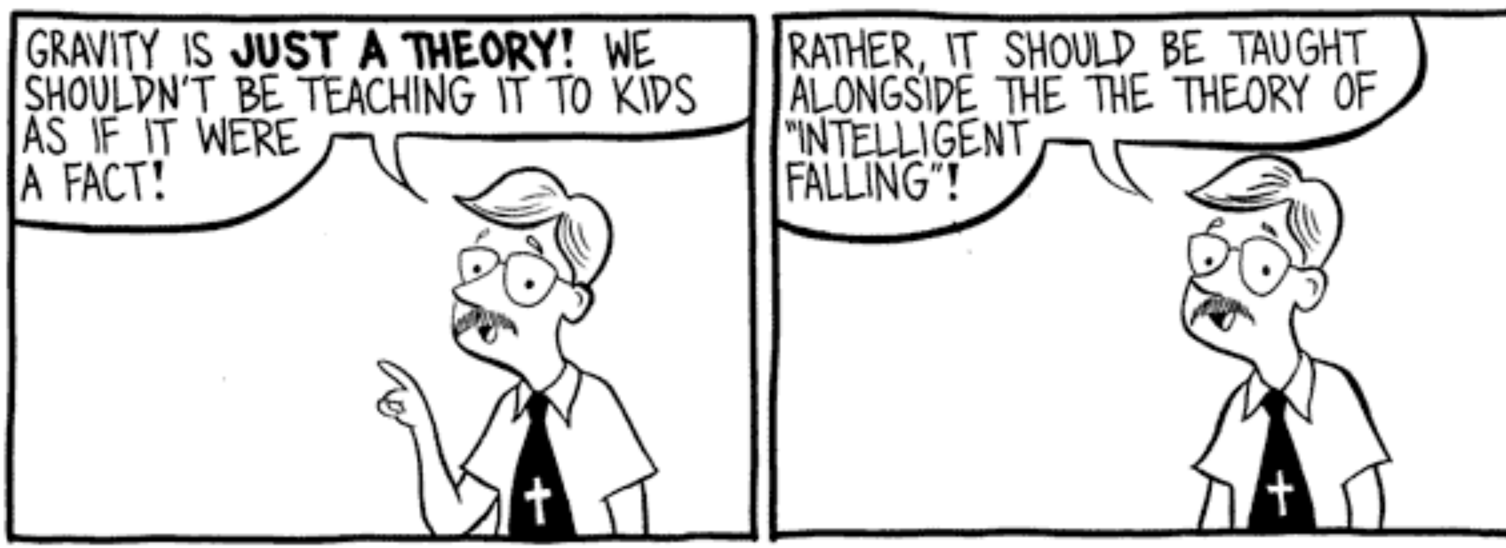
Setting $\delta S = 0$ we obtain the Einstein equation

$$R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R + \Lambda g^{\mu\nu} = 8\pi G T^{\mu\nu}\,. \qquad (5)$$

Now I do not regret to have introduced this term. **by**




Covariant Galileons
Randall-Sundrum
Horndeski
K-inflation
Degravitation
Bimetric theories
General Scalar-Tensor
f(G)
Einsten-Aether
Kinetic Gravity Braiding
Massive Gravity
Horava-Lifschitz
Cascading gravity
TeVeS
Brans-Dicke
Fab Four
EBI gravity


GRAVITY IS **JUST A THEORY!** WE SHOULDN'T BE TEACHING IT TO KIDS AS IF IT WERE A FACT!
RATHER, IT SHOULD BE TAUGHT ALONGSIDE THE THE THEORY OF "INTELLIGENT FALLING"!

Parameterisations

- How useful?

convenient → put into a modified Einstein-boltzmann solver.
Doesn't cover the full theory space.
Even a smoking gun detection won't necessary lead to a particular theory.
Is there a smoking gun? E.g. could clustering dark energy mimic any signature.

$$k^2\Psi = -\mu(k,a)4\pi G a^2\{\rho\Delta + 3(\rho+P)\sigma\}$$

$$k^2[\Phi - \gamma(k,a)\Psi] = \mu(k,a)12\pi G a^2(\rho+P)\sigma$$

$$\mu(k,a) = \frac{1+\frac{4}{3}\lambda_1^2\,k^2a^s}{1+\lambda_1^2\,k^2a^s} \qquad \gamma(k,a) = \frac{1+\frac{2}{3}\lambda_1^2\,k^2a^s}{1+\frac{4}{3}\lambda_1^2\,k^2a^s}$$

- E.g. eqn 2 params related to a particular theory.
- Works in the quasi-static newtonian regime (what about super-horizon/non-linear).

- move on to **PPF**