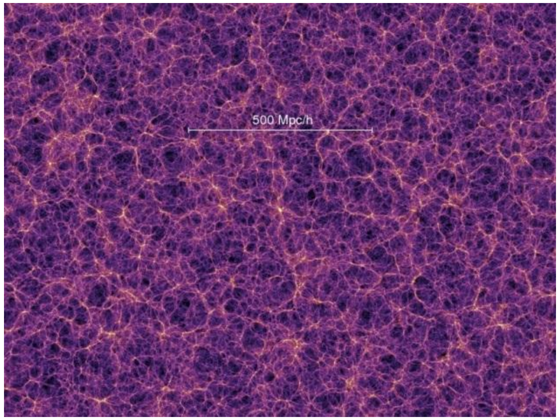
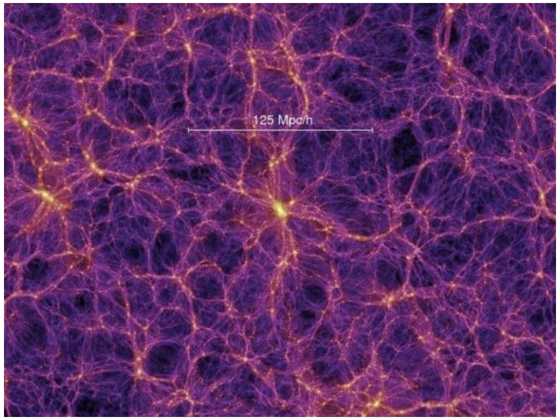
Inhomogeneity in Cosmology

Timothy Clifton
(Queen Mary, University of London)

The inhomogeneous universe

The late universe appears homogeneous on large scales, but is very inhomogeneous on small scales:





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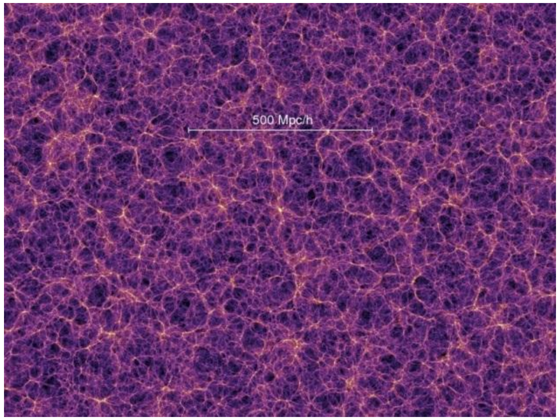

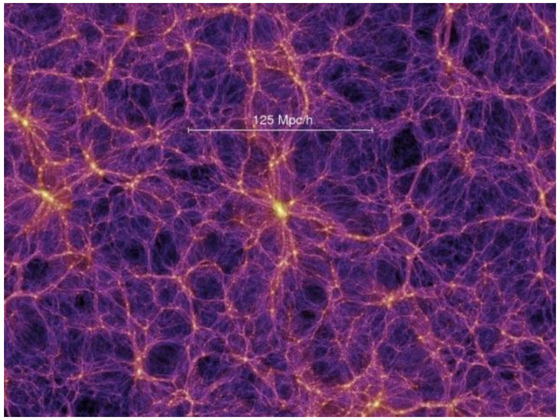

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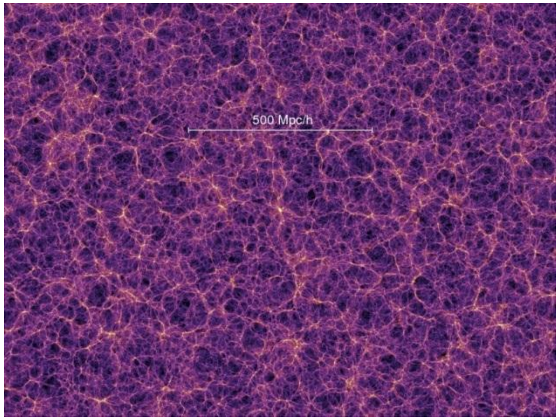

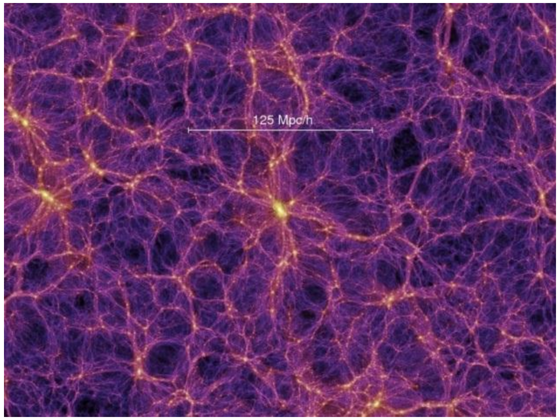

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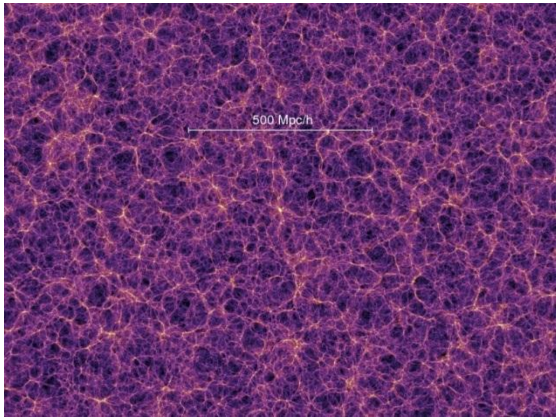

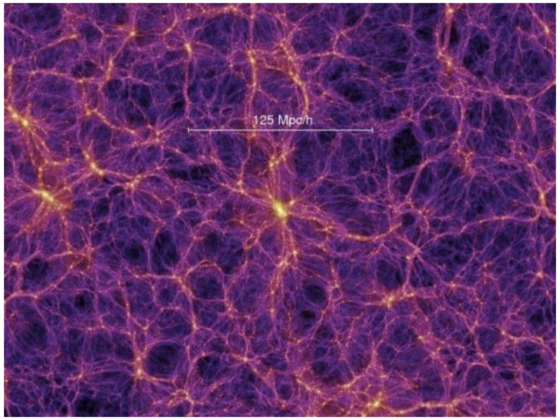

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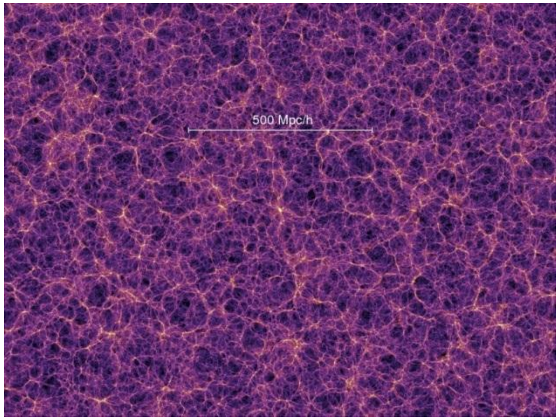

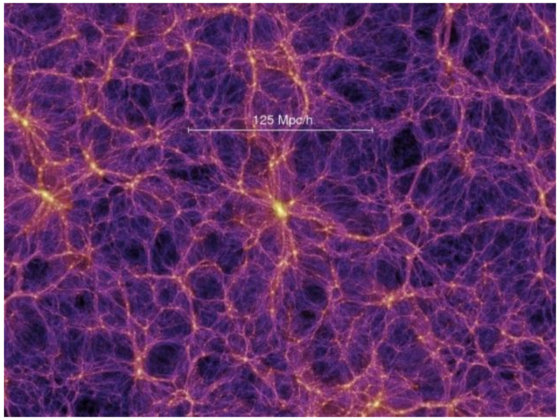

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- How to link observations to the large-scale evolution.

  - scale dependence? average of null geodesics?

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(without assuming what we are trying to investigate).

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- Fit observations to an FRW geometry.
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- Construct new cosmological models that do not use averaging, or that do not assume the large-scale evolution of the universe from the outset.

  - bottom-up approaches

- …

Buchert's equations

- Define an 'average' scale factor: $$a_{\mathcal{D}}(t) \equiv \left(\frac{\int_{\mathcal{D}} d^3X \sqrt{^{(3)}g(t, X^i)}}{\int_{\mathcal{D}} d^3X \sqrt{^{(3)}g(t_0, X^i)}} \right)^{\frac{1}{3}}$$

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- Average the Hamiltonian, Raychaudhuri & conservation eqns:

$$3\frac{\dot{a}_{\mathcal{D}}^2}{a_{\mathcal{D}}^2} = 8\pi G_{\rm N} \langle \rho \rangle_{\mathcal{D}} - \frac{1}{2}\langle {}^{(3)}R \rangle_{\mathcal{D}} - \frac{1}{2}\mathcal{Q}_{\mathcal{D}}$$

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$$\partial_t \langle \rho \rangle_{\mathcal{D}} + 3\frac{\dot{a}_{\mathcal{D}}}{a_{\mathcal{D}}} \langle \rho \rangle_{\mathcal{D}} = 0$$

where: $\langle \psi \rangle_{\mathcal{D}}(t) \equiv \frac{\int_{\mathcal{D}} d^3X \sqrt{^{(3)}g(t, X^i)}\, \psi(t, X^i)}{\int_{\mathcal{D}} d^3X \sqrt{^{(3)}g(t, X^i)}}$ $\quad \mathcal{Q}_{\mathcal{D}} \equiv \frac{2}{3}\left(\langle \Theta^2 \rangle_{\mathcal{D}} - \langle \Theta \rangle_{\mathcal{D}}^2 \right) - 2\langle \sigma^2 \rangle_{\mathcal{D}}$

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foliation dependent

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← not a closed system

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applicable to scalars only

Observables

- The monopole from the Kristian-Sachs $d_L(z)$ gives:

$$q_0 = \frac{1}{H_0^2}\left[\frac{4\pi G}{3}\left(\rho + 3p + 12\sigma^2\right) - \frac{\Lambda}{3}\right]_0$$

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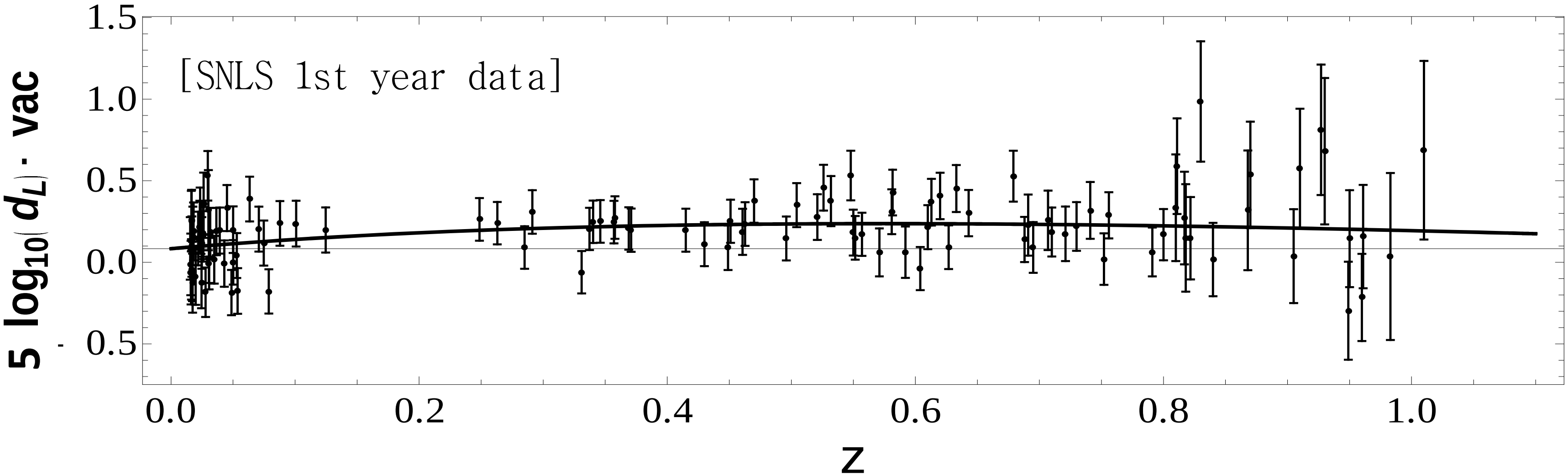
- Supernova observations imply acceleration based on complicated fitting procedures:



Recent results

- In spacetimes with a homogeneity scale, Buchert's equations can closely follow the mean value of large-scale observations.

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- If back-reaction is responsible for the supernova observations, then Clarkson's and Buchert's measures of acceleration should be expected to be very different.
  [Bull & Clifton, PRD 85, 103512 (2012)]

Zalaletdinov's equations

- Define: $$\langle p^{\alpha\ldots}_{\beta\ldots}(x)\rangle = \frac{1}{V_\Sigma}\int_\Sigma \sqrt{-g'}d^4x' p^{\mu'\ldots}_{\nu'\ldots}(x')\mathcal{A}^{\alpha}{}_{\mu'}(x,x')\mathcal{A}^{\nu'}{}_{\beta}(x,x')\ldots$$

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- To get:

$$\boxed{\bar{g}^{\beta\epsilon}M_{\gamma\beta} - \frac{1}{2}\delta^{\epsilon}{}_{\gamma}\bar{g}^{\mu\nu}M_{\mu\nu} = 8\pi G\bar{T}^{\epsilon}{}_{\gamma} - (Z^{\epsilon}{}_{\mu\nu\gamma} - \frac{1}{2}\delta^{\epsilon}{}_{\gamma}Q_{\mu\nu})\bar{g}^{\mu\nu}}$$

[Zalaletdinov, Bull. Astron. Soc. India 25, 401 (1997)]

where: $$M^{\mu}{}_{\nu\alpha\beta} = \partial_\alpha\langle\Gamma^{\mu}{}_{\nu\beta}\rangle - \partial_\beta\langle\Gamma^{\mu}{}_{\nu\alpha}\rangle + \langle\Gamma^{\mu}{}_{\sigma\alpha}\rangle\langle\Gamma^{\sigma}{}_{\nu\beta}\rangle - \langle\Gamma^{\mu}{}_{\sigma\beta}\rangle\langle\Gamma^{\sigma}{}_{\nu\alpha}\rangle$$

$$Z^{\alpha}{}_{\beta\gamma}{}^{\mu}{}_{\nu\sigma} = \langle\Gamma^{\alpha}{}_{\beta[\gamma}\Gamma^{\mu}{}_{\underline{\nu}\sigma]}\rangle - \langle\Gamma^{\alpha}{}_{\beta[\gamma}\rangle\langle\Gamma^{\mu}{}_{\underline{\nu}\sigma]}\rangle$$

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$$Q_{\mu\nu} = Z^{\alpha}{}_{\mu\nu\alpha}$$

obeys its own conservation and constraint equations

Solutions

- The FLRW solutions to Zalaletdinov's equations are:

$$ds^2 = \langle g_{\mu\nu} \rangle \, dx^\mu dx^\nu = -dt^2 + a^2(t) \left[\frac{dr^2}{1 - k_{\mathrm{g}} r^2} + r^2 d\Omega \right]$$

where: $$H^2 = \frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}\rho - \frac{k_{\mathrm{d}}}{a^2} + \frac{\Lambda}{3} \; .$$

[Coley, Pelavas & Zalaletdinov, PRL 95, 151102 (2005)]
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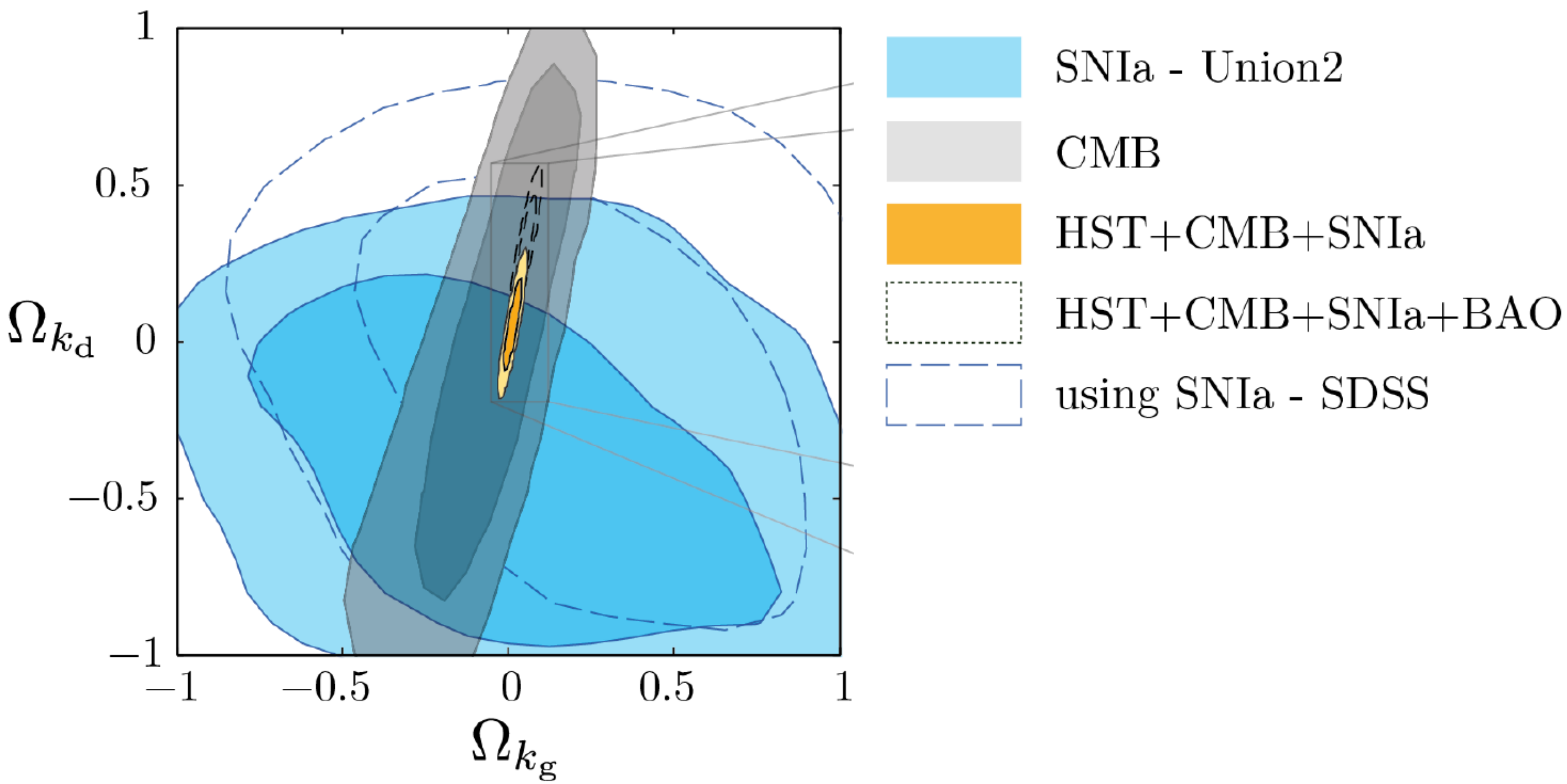
- Luminosity distances calculated within this geometry are:

$$d_L(z) = \frac{(1+z)}{H_0 \sqrt{|\Omega_{k_{\rm g}}|}} f_{k_{\rm g}} \left(\int_{\frac{1}{1+z}}^{1} \frac{\sqrt{|\Omega_{k_{\rm g}}|} da}{\sqrt{\Omega_{k_{\rm d}} a^2 + \Omega_\Lambda a^4 + \Omega_m a}} \right)$$

[Clarkson, Clifton, Coley, Sung, PRD 85, 043506 (2012)]
see also [Clifton, Coley, van den Hoogen, JCAP 10, 044 (2012)]

Comparing with observations



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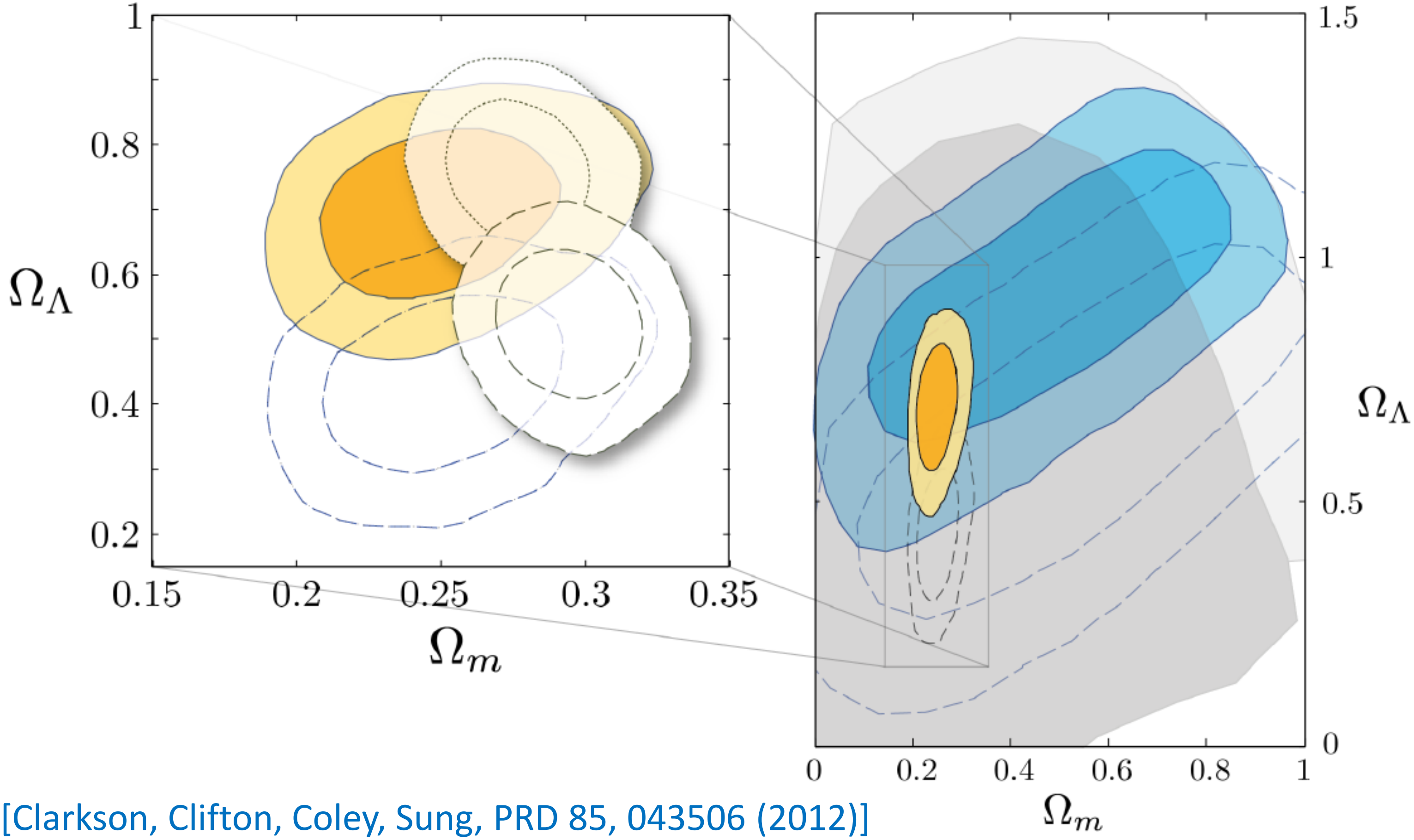

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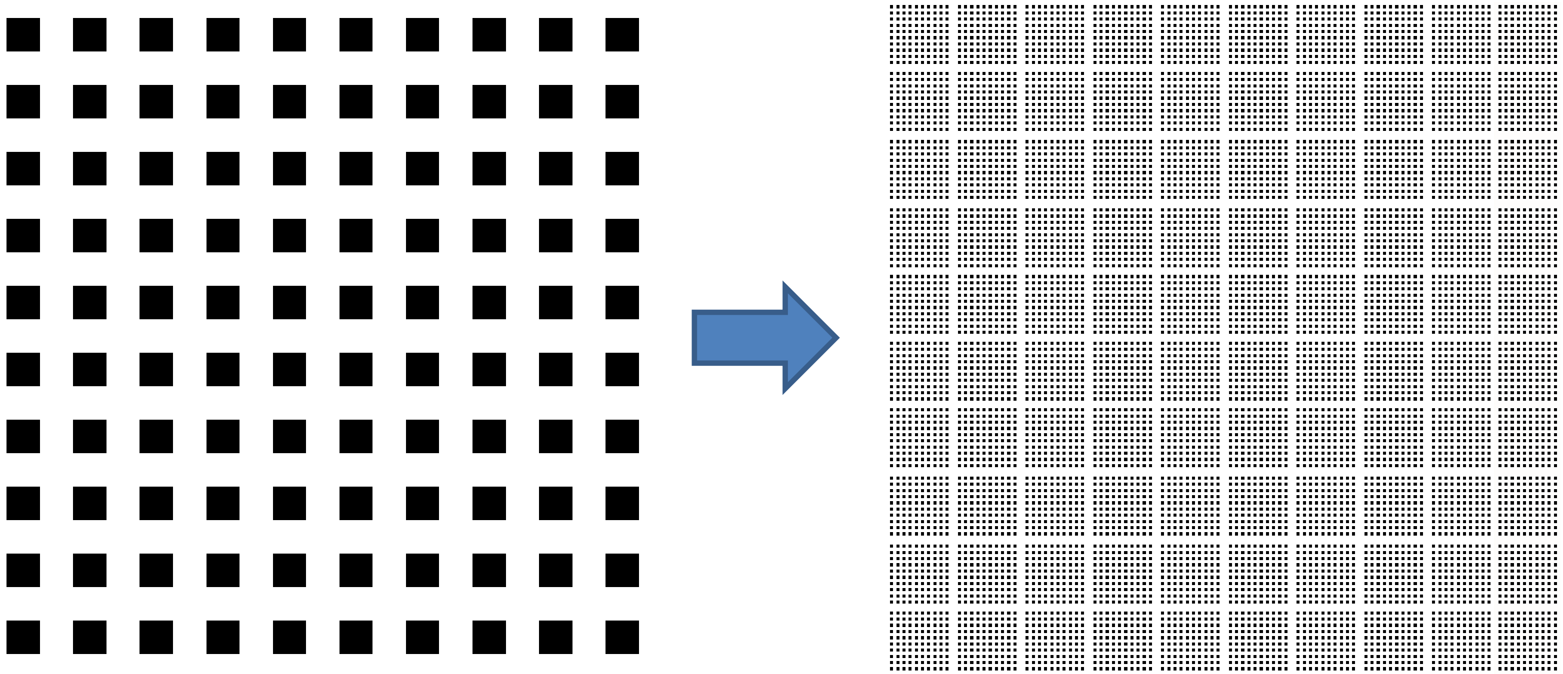
Cosmology without averaging

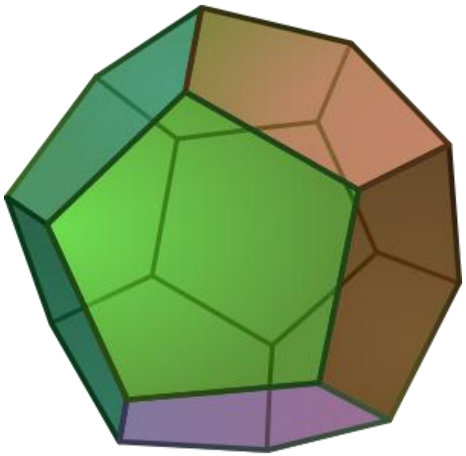
- Instead of averaging we can try to construct a universe from discrete masses, and compare it to FLRW:



Tiling a closed space

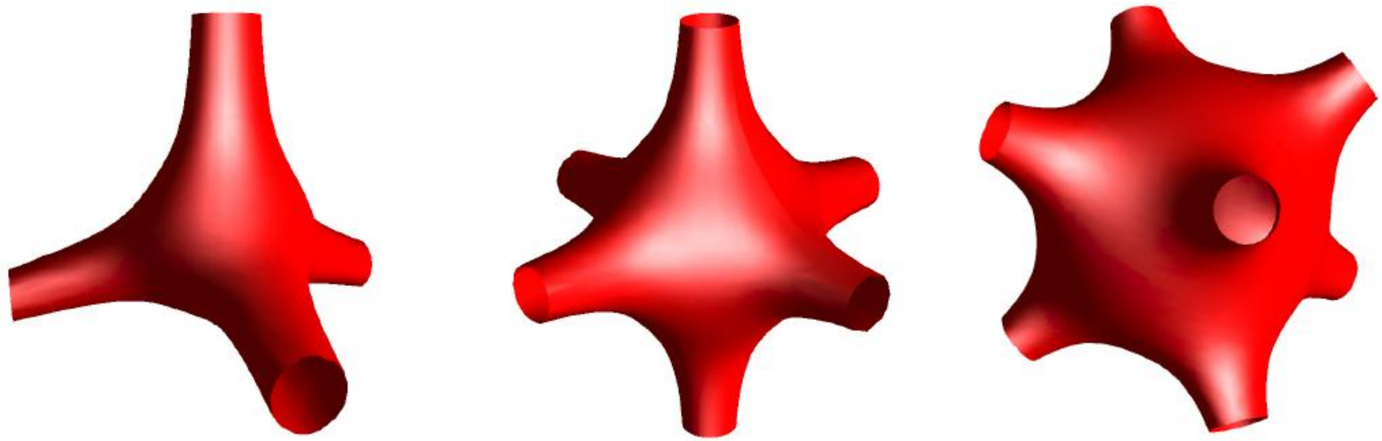
- There are six possible tilings of a closed space with regular polyhedra, and one with two balls:

Lattice Structure	Cell Shape	Number of Cells
-	Ball	2
{333}	Tetrahedron	5
{433}	Cube	8
{334}	Tetrahedron	16
{343}	Octahedron	24
{533}	Dodecahedron	120
{335}	Tetrahedron	600

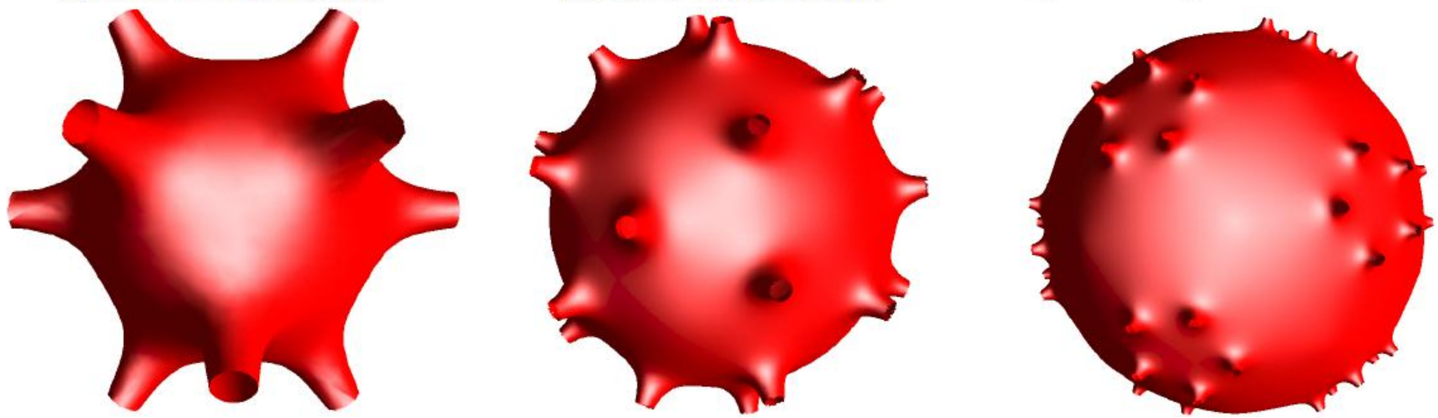


Exact solutions

- At the maximum of expansion the geometry can be found exactly:



(a) A slice through the 5-cell model (b) A slice through the 8-cell model (c) A slice through the 16-cell model

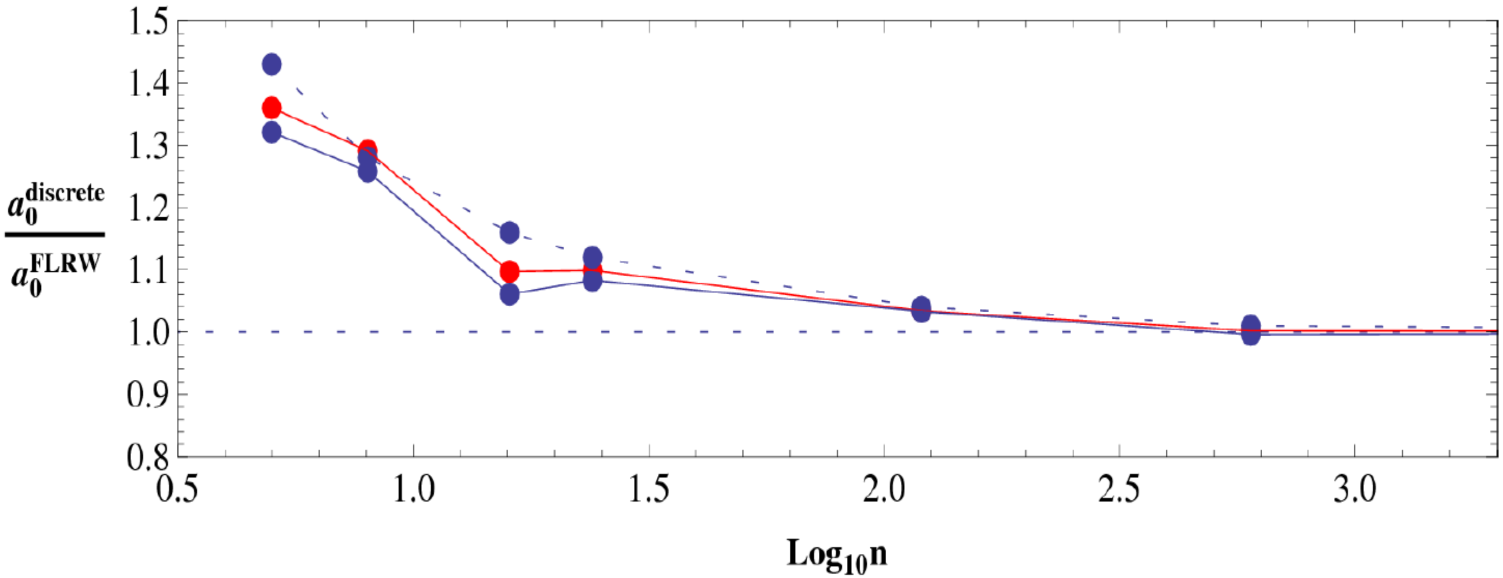


(d) A slice through the 24-cell model (e) A slice through the 120-cell model (f) A slice through the 600-cell model

[Clifton, Rosquist & Tavakol, arXiv:1203.6478 (2012)]

Comparing to FLRW

- Comparing to a spacetime with smoothly distributed mass:



[Lindquist & Wheeler, Rev. Mod. Phys. 29, 432 (1957)]
[Clifton & Ferreira, PRD 80, 103503 (2009)]
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Discussion and outlook

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- Understanding the large-scale behaviour of inhomogeneous spacetimes is crucial for evaluating the evidence for dark energy.

- Back-reaction must be quantified in order to undertake precision cosmology.

- More sophisticated models and methods may be required to make further progress on these problems.

Thank you