Testing Modified Gravity in the Solar System

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1204.6663 [Magueijo, **AM**]
1212.3905 [**AM**]

Introduction

The two cornerstones of modern cosmology are General Relatvity and the ΛCDM model. We see from the Einstein equations:

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}R\, g_{\mu}\nu + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

On their own they allow us to map the behaviour of the early universe from today to the distant past, as well as use Newtonian and other methods to describe the various energy and matter densities of the early universe, culminating in the picture at last scattering, the CMB, as well as the power spectrum and much much more...

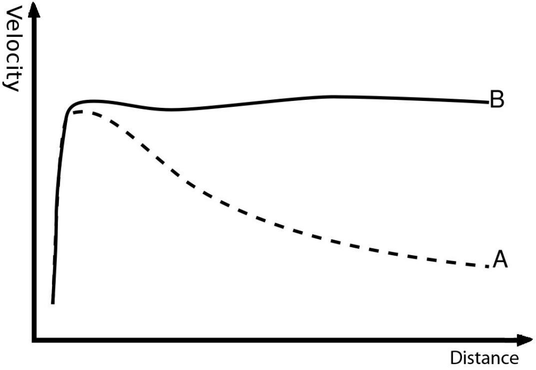
But each is not without its problems... Maybe we can do better?

If this is the case, how hard do scientists have to work to reach the *next generation* of results to rule in or out these ideas?

A Few Problems

GR however on its own has issues, including:

1. The Tully - Fisher relation: Relationship between the luminosity of galaxies and their rotation velocities, $L \propto v^4$
2. Flat galaxy rotation curves:



A Few Solutions

1. Modifying the matter, introducing a non-luminous matter component, so called **dark matter**. Galaxies with halos of such dark matter can explain these anomalous dynamics...
   Pros: Provides a decent fit to galactic data, formation and evolution simulations, as well as helping early universe data, ... **Cons:** No candidate dark matter particle has been conclusively detected...

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2. Modifying the underlying dynamics, introducing an acceleration dependent theory of **modified gravity**, bringing about these observed effects at typical galactic acceleration scales, $a_0 \simeq 10^{-10} ms^{-2}$, whilst reducing to the usual Newtonian/GR dynamics on larger scales.
   Pros: Empirically simple and not exceptionally difficult to make such a limit appear from a fully relativistic theory... **Cons:** Many "free parameters", some fine tuning required...

Lets play around with the dynamics... MOND

Modified Newtonian Dynamics (MOND) was first suggested by Milgrom, in the early 1980s, as a way of explaining the flat galaxy rotation curves. His original formulation reframes the Newtonian force law:

$$\mathbf{F}^{(N)} = m\,\mathbf{a} \quad \longrightarrow \quad \mathbf{F}^{(N)} = m\tilde{\mu}\left(\frac{|\mathbf{a}|}{a_0}\right)\mathbf{a}$$

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$$\mathbf{F}^{(N)} = m\,\mathbf{a} \quad \longrightarrow \quad \mathbf{F}^{(N)} = m\tilde{\mu}\left(\frac{|\mathbf{a}|}{a_0}\right)\mathbf{a}$$

$$F^{(N)} = \frac{GMm}{r^2} \qquad\qquad a = \frac{v^2}{r}$$

$$\tilde{\mu}\,(x) \longrightarrow \begin{cases} 1 & x \gg 1 \\ x & x \ll 1 \end{cases} \quad \Longrightarrow \quad F \longrightarrow \begin{cases} F^{(N)} & a \gg a_0 \\ \sqrt{F^{(N)} a_0} & a \ll a_0 \end{cases}$$

Low Acceleration Targets in the Solar System

Consider a two-body gravitational system, masses M and m, $M \gg m$ with separation R between them. Along the line linking the two bodies we find the force given as

$$\mathbf{F}^{(\mathbf{N})} = \left(-\frac{GMm}{r^2} + \frac{GMm}{(r-R)^2} \right) \mathbf{e_z}$$

The saddle point (SP) of the gravitational potential is then located at

$$r = r_s \approx R \left(1 - \sqrt{\frac{m}{M}} \right)$$

Around this point, the force is linearised as

$$\mathbf{F}^{(N)} \approx A(r - r_s)\mathbf{e_z}$$

But this is all for a strictly Newtonian case, how does MOND compare?

Saddle Point Science

Taking the rule of thumb for $a \leq a_0$,

$$F \approx \sqrt{F^{(N)} a_0}$$

The "near" SP regime now looks like

$$F \approx \pm\sqrt{A a_0 |r - r_s|}$$

Trouble is the tidal stresses $\partial F_i / \partial x_j$ now appear infinite as $r \rightarrow r_s$!!

Perhaps something is amiss here - but it does suggest at least a proof of concept MOND test...

A Test Case: TeVeS

Lets pick a modified gravity theory with a preferred acceleration scale:

$$S = \frac{1}{2}\int d^4x\sqrt{-g}\left(\frac{1}{8\pi G}R^{(4)} - \frac{\mu}{\kappa G}h^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}\frac{\mu^2}{\ell^2\kappa^2 G}F(\mu) - \frac{1}{16\pi G}\left\{K\mathcal{F}^{\alpha\beta}\mathcal{F}_{\alpha\beta} - 2\lambda\left(A_\mu A^\mu + 1\right)\right\}\right) + \int d^4x\sqrt{-\tilde{g}}\,\mathcal{L}(\tilde{g}_{\mu\nu}, f^\alpha)$$

where: κ, K are the coupling constants for the scalar and vector fields
$\ell \equiv \sqrt{3\kappa}/4\pi a_0$
μ is a non-dynamical scalar field
$h^{\mu\nu} \equiv g^{\mu\nu}A^\mu A^\nu$
$\mathcal{F}_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$
$F(\mu)$ is a free function

In short a **real** mess, but take the usual NR limit and we find....

NR TeVeS

The *physical* gravitational field is split up as

$$\nabla\Phi = \nabla\Phi_N + \nabla\phi$$

with ϕ having its own dynamics governed by

$$\nabla \cdot \left[\mu\left(\frac{\kappa}{4\pi}\frac{|\nabla\phi|}{a_0}\right)\nabla\phi\right] = \kappa G\rho$$

$$\mu(z) \longrightarrow \begin{cases} 1 & z \gg 1 \\ z & z \ll 1 \end{cases} \Longrightarrow F \longrightarrow \begin{cases} F^{(N)} + \frac{\kappa}{4\pi}F^{(N)} & a \gg a_0 \\ F^{(N)} + \sqrt{F^{(N)}a_0} & a \leq a_0 \end{cases}$$

NR TeVeS

We pick a μ, for the moment use

$$\frac{\mu}{\sqrt{1-\mu^4}} = z\,, \quad z \ll 1 \quad \mu \to z + \dots$$

$$z \gg 1 \quad \mu \to 1 - \frac{1}{4z^2} + \dots$$

(Choice seems arbitrary, picked to aid analytical progression).
Linearise the equations with a change of variable

$$\mathbf{U} \equiv -\mu \frac{\kappa}{4\pi} \frac{\nabla\phi}{a_0} \quad \Rightarrow \quad \nabla \cdot \mathbf{U} = \#\rho$$

$$U = \mu z \quad \Rightarrow \quad \mu = \frac{U^{1/2}}{(1+U^2)^{1/4}}$$

U Formalism

$$\mathbf{U} \equiv -\mu \frac{\kappa}{4\pi} \frac{\nabla \phi}{a_0}$$

$$\nabla \cdot (\mu \nabla \phi) = \kappa G \rho \Rightarrow \nabla \cdot \mathbf{U} = 0|_{SP}$$

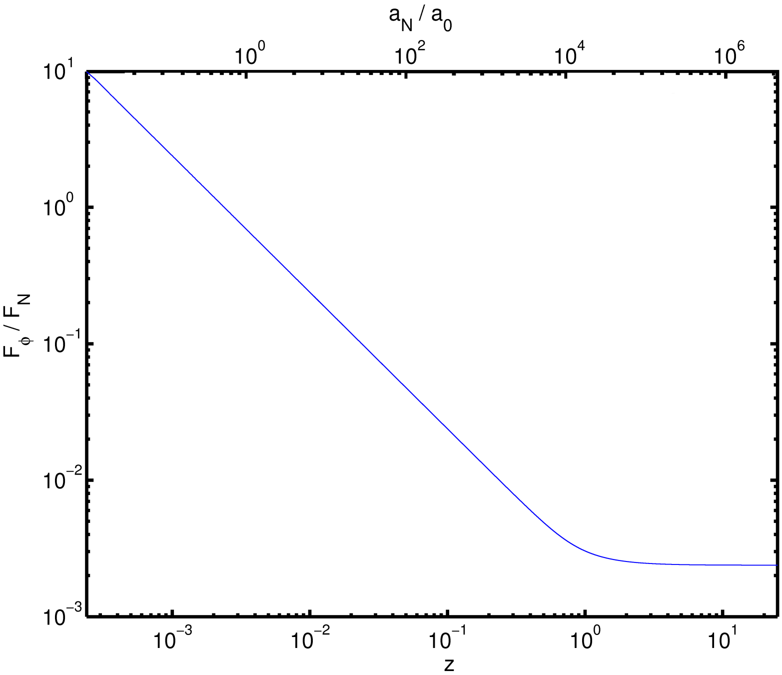
$$\nabla \wedge \left(\frac{\mathbf{U}}{\mu} \right) = 0 \Rightarrow 4m\, U^2 \nabla \wedge \mathbf{U} + \mathbf{U} \wedge \nabla U^2 = 0$$

$$4m = \frac{d \ln U^2}{d \ln \mu} = 4(1 + U^2)$$

We find the MONDian force from

$$\delta \mathbf{F} = -\nabla \phi = \frac{4\pi a_0}{\kappa} \frac{\mathbf{U}}{\mu} = \frac{4\pi a_0}{\kappa} \mathbf{U} \left(1 + \frac{1}{U^2} \right)^{1/4}$$

Find $\mathbf{U}$ dynamics $\Rightarrow$ Find $\nabla \phi$ dynamics...


a_N / a_0
10^0
10^2
10^4
10^6
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
F_ϕ / F_N
10^{-3}
10^{-2}
10^{-1}
10^0
10^1
z

Different Regimes

The form of behaviour of $\mathbf{U}$ is determined by the particular regime we are looking, i.e. the deep MOND $|\mathbf{U}| \ll 1$ or the quasi-Newtonian $|\mathbf{U}| \gg 1$.

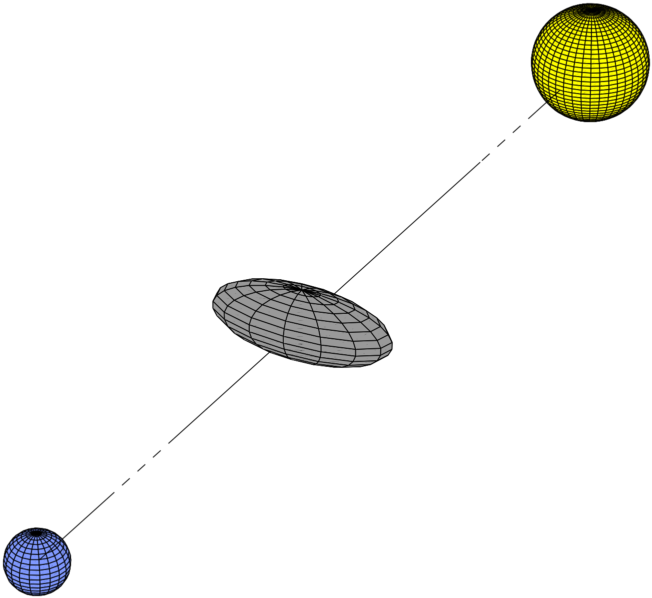
Separating the two regimes is $|\mathbf{U}|^2 \simeq 1$

This corresponds to an ellipsoid with semi-major axis of size

$$r_0 = \left(\frac{16\pi^2 a_0}{\kappa^2 A}\right)$$

For the Earth-Sun system, $r_0 \approx 383$ km!

For the Jupiter-Sun system, $r_0 \approx 10^6$ km!!



Different Regime Observables

$$\nabla \cdot \mathbf{U} = 0 \qquad 4(1+U^2)\, U^2 \nabla \wedge \mathbf{U} + \mathbf{U} \wedge \nabla U^2 = 0$$

Quasi-Newtonian, $\mathbf{U} = \frac{r}{r_0}\mathbf{N}(\psi) + \frac{r_0}{r}\mathbf{B}(\psi)$

$$\delta \mathbf{F} = -\nabla \phi = \frac{4\pi a_0}{\kappa}\frac{\mathbf{U}}{\mu} \simeq \frac{4\pi a_0}{\kappa}\left(\underbrace{\frac{r}{r_0}\mathbf{N}}_{G_N \text{ renorm}} + \underbrace{\frac{r_0}{r}\left(\frac{\mathbf{N}}{4N^2} + \mathbf{B}\right)}_{\text{main observable}} + \dots \right)$$

Different Regime Observables

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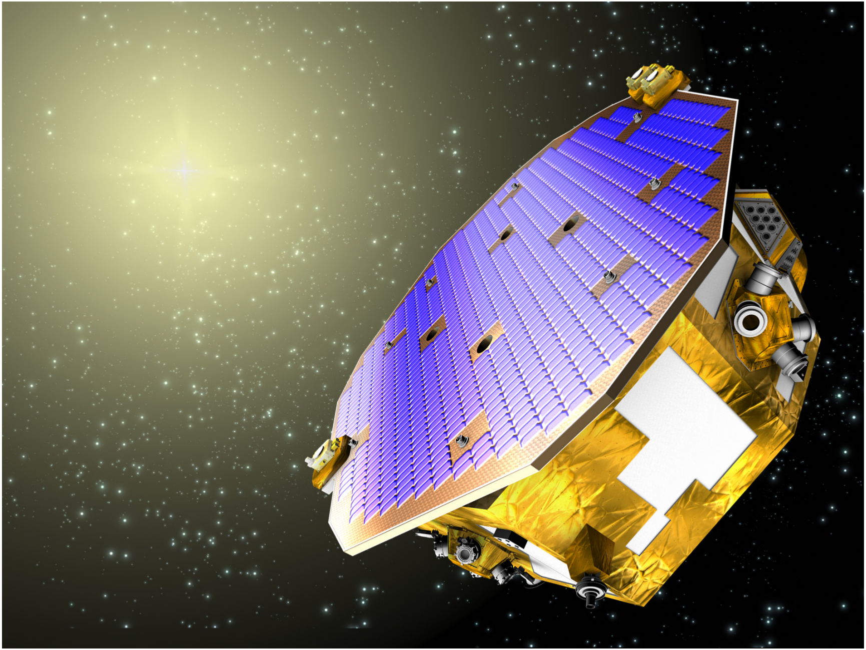
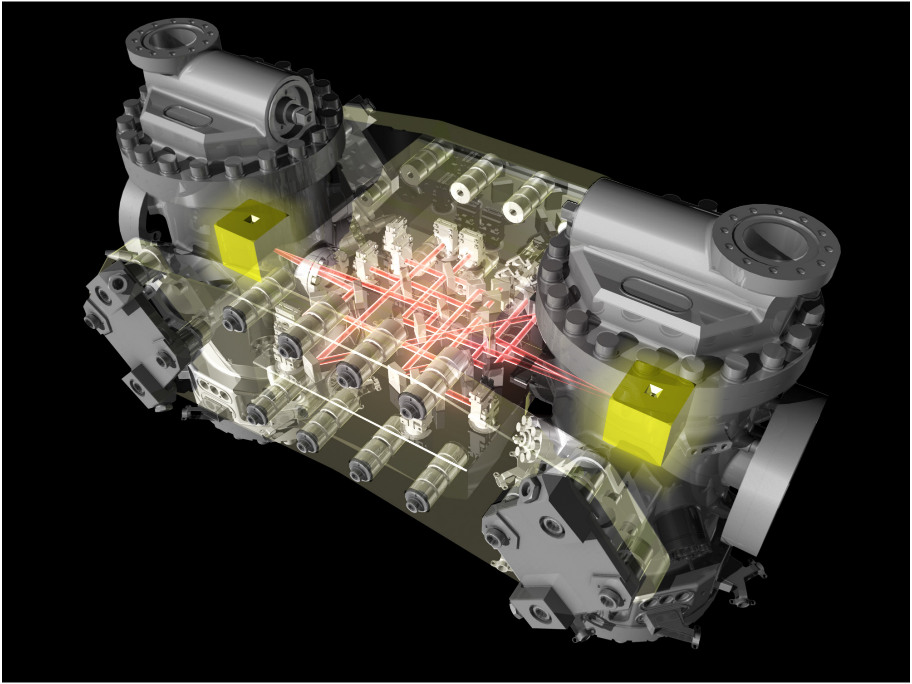
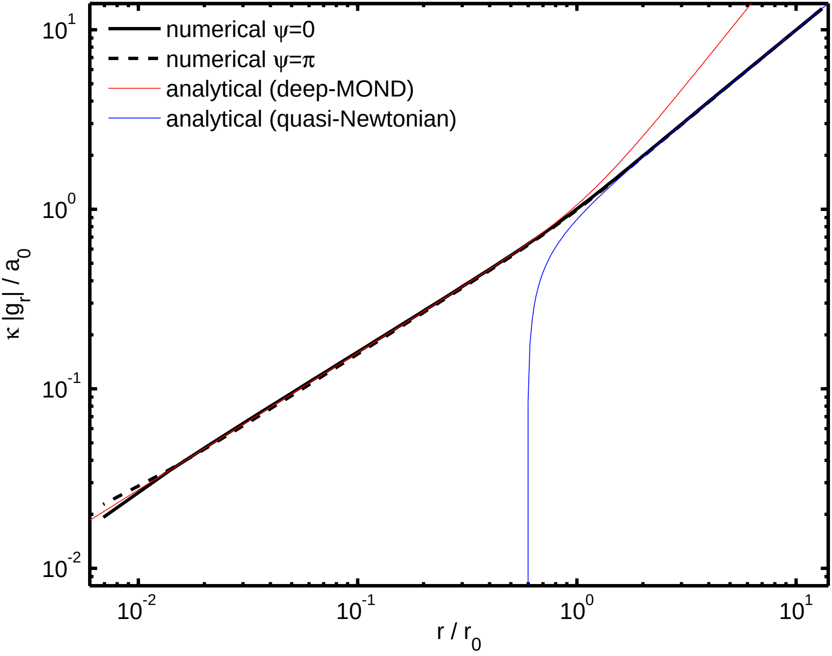
Deep MOND, $\mathbf{U} = C\left(\frac{r}{r_0}\right)^{\alpha-2}\mathbf{D}(\psi)$

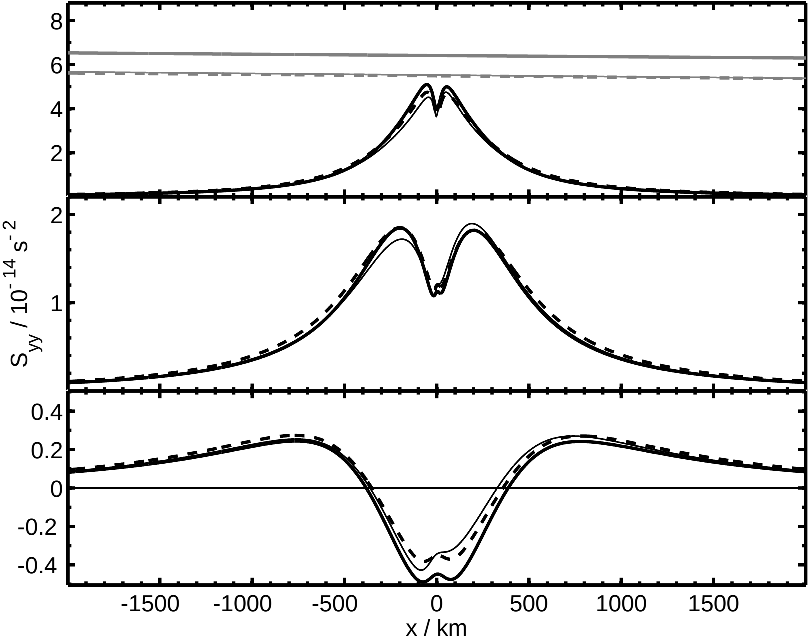
$$\delta \mathbf{F} = -\nabla\phi = \frac{4\pi a_0}{\kappa}\frac{\mathbf{U}}{\mu} \simeq \frac{4\pi a_0}{\kappa} C^{1/2}\left(\frac{r}{r_0}\right)^{\frac{\alpha-2}{2}}\frac{\mathbf{D}}{D^{1/2}}$$

where $C \approx 0.84$, $\alpha \approx 3.528$ and $\mathbf{D}(\psi) \simeq \mathbf{N}(\psi)$

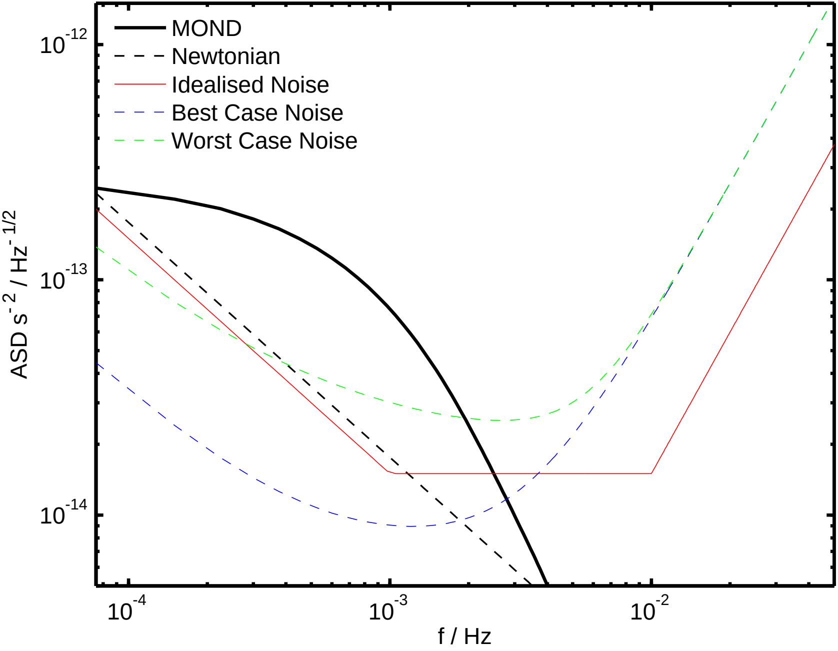





numerical ψ=0
numerical ψ=π
analytical (deep-MOND)
analytical (quasi-Newtonian)
κ |g_r| / a_0
r / r_0
10^1
10^0
10^-1
10^-2
10^-2
10^-1
10^0
10^1



8
6
4
2
2
1
0.4
0.2
0
-0.2
-0.4
S_{yy} / 10^{-14} s^{-2}
-1500
-1000
-500
0
500
1000
1500
x / km


MOND
Newtonian
Idealised Noise
Best Case Noise
Worst Case Noise
10^{-12}
10^{-13}
10^{-14}
ASD s^{-2} / $Hz^{-1/2}$
10^{-4}
10^{-3}
10^{-2}
f / Hz

Signal to Noise Ratio (SNR)

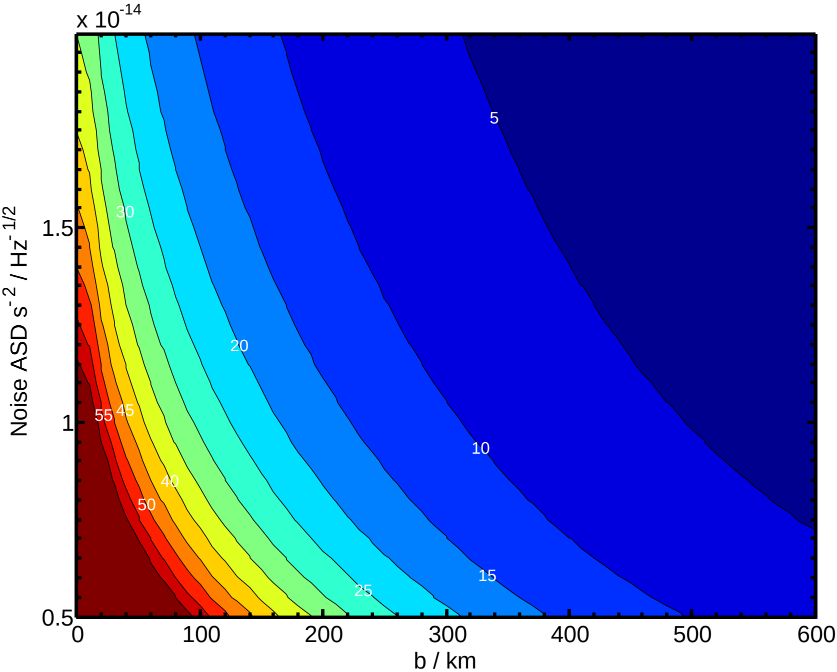
Use noise matching filtering from GW searches, find optimal SNR from observable tidal stress signal:

$$
\begin{aligned}
S_{ij}(\mathbf{x}) &= -\frac{\partial^2\phi}{dx_i dx_j} + \frac{\kappa}{4\pi}\frac{\partial^2\Phi^N}{dx_i dx_j} \\
S_{ij}(\mathbf{x}) &\rightarrow S_{ij}(\mathbf{v}t) \rightarrow \tilde{S}(f)
\end{aligned}
$$

For a "typical" trajectory, $b = 50$km closest approach run, velocity $v \simeq 1.5kms^{-1}$ and idealised noise, **SNR** $\simeq 28$...

Orbit designers tell us *much* closer is now within reach...

$$
SNR_{opt} = 2\sqrt{\int_0^\infty \frac{|\tilde{S}(f)|^2}{\tilde{S}_n(f)} df}
$$


x 10^-14
Noise ASD s^-2 / Hz^-1/2
1.5
1
0.5
0
100
200
300
400
500
600
b / km
5
10
15
20
25
30
40
45
50
55

Constraints

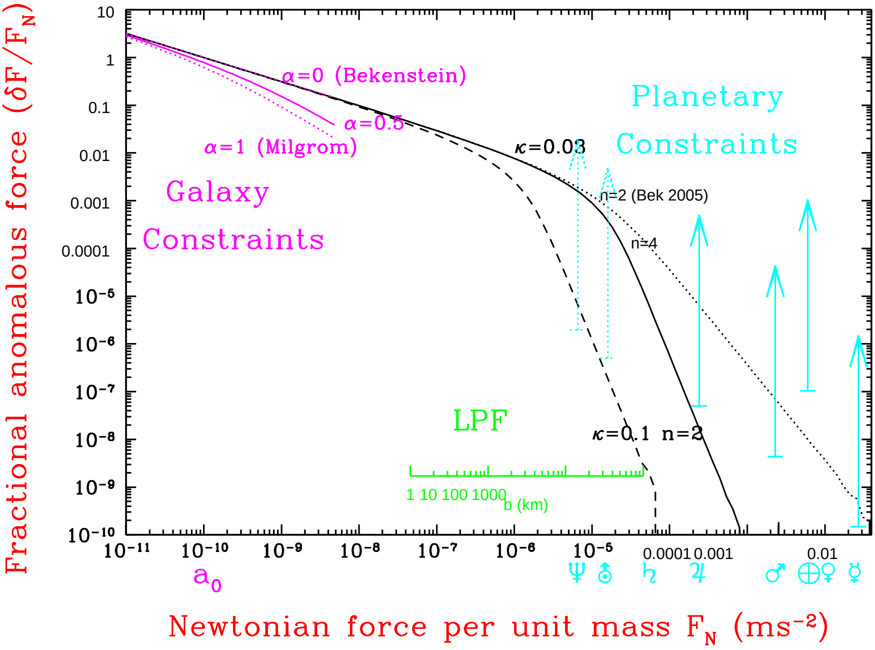
Within these theories, there are a few free parameters - κ, a_0 and μ

Galactic DM alternative $\Rightarrow a_0 \simeq 10^{-10} ms^{-2}$, but if these theories are just alternatives to GR, then allow for greater freedom...

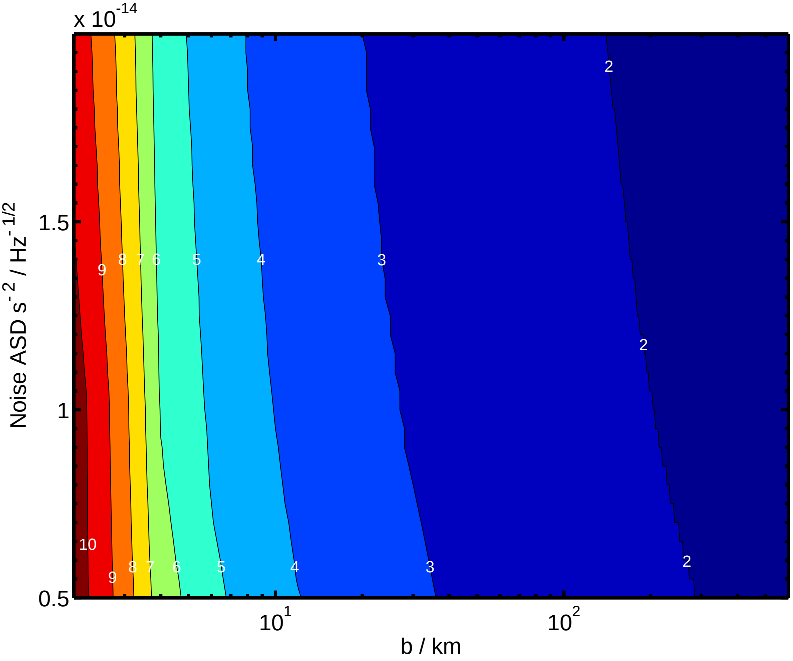
- G_N **Renomalisation**, Fix from Cosmology

$$G_{ren} = G_N \left(1 + \frac{\kappa}{4\pi}\right)$$

- **Designer μ's**, constrain from null results and strong constraints on anomalous accelerations within Solar System...

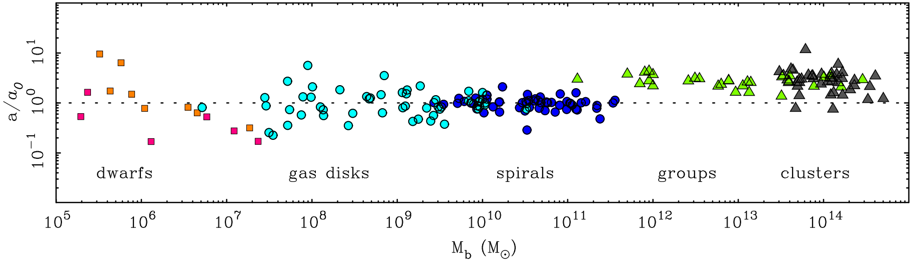


Fractional anomalous force (δF/F_N)
Newtonian force per unit mass F_N (ms^-2)
α=0 (Bekenstein)
α=0.5
α=1 (Milgrom)
Galaxy
Constraints
Planetary
Constraints
κ=0.03
n=2 (Bek 2005)
n=4
κ=0.1 n=2
LPF
1 10 100 1000
b (km)
a_0


x 10^-14
Noise ASD s^-2 / Hz^-1/2
1.5
1
0.5
9
8
7
6
5
4
3
2
2
10
9
8
7
6
5
4
3
2
10^1
10^2
b / km

Free Variables?

How much variation can we allow in our parameters without recomputing all of our results again?
- $\kappa \sim \mathcal{O}(10^{-2})$ to allow for structure formation
- $a_0 \simeq 10^{-10} ms^{-2}$ from galactic constraints, however astrophysics is rarely clean...



(1112.3960)

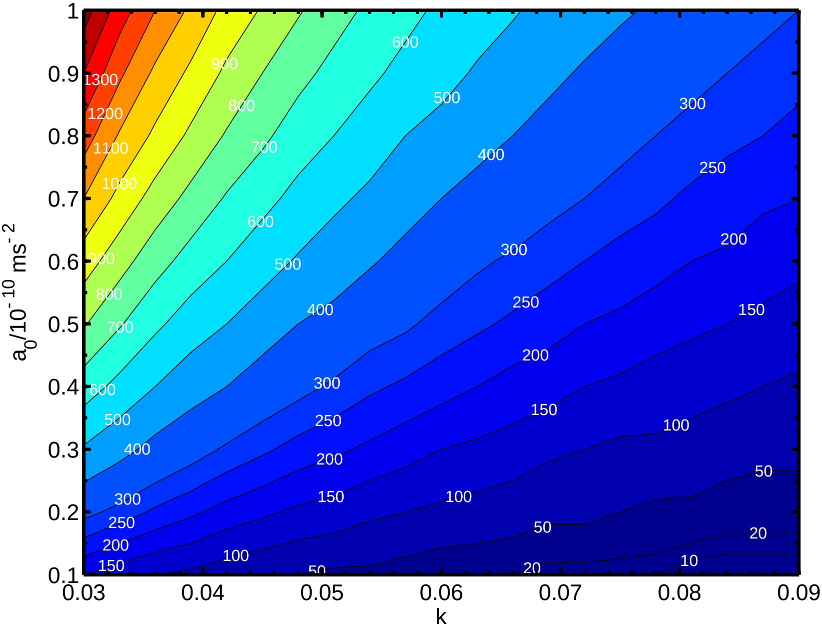
Rescaling a_0 and κ

We find from vacuum MONDian field equations have an inherent scale invariance in the spatial variables, hence we have tidal stresses

$$S_{ij}^M = \kappa A H_{ij}\left(\frac{\mathbf{x}}{r_0}\right)$$
$$r_0 = \frac{16\pi^2}{A}\frac{a_0}{\kappa^2}$$

So we just need to rescale our current results for varying κ, a_0!

$\kappa^{(0)} = 0.03$ and $a_0^{(0)} = 10^{-10} ms^{-2}$ are our fiducial values and quantify the effect of changing these on the SNR as well as constraining this variation in the case of a null result...



$a_0/10^{-10}$ ms^{-2}
k
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0.03
0.04
0.05
0.06
0.07
0.08
0.09
1300
1200
1100
1000
900
800
700
600
500
400
300
250
200
150
100
50
20
10

Looking beyond null results and rescaling constants

In the event of LPF showing *anything* above the Newtonian background, what (if any) conclusions can we make?

- How (if at all) can we constrain the parameters present here?

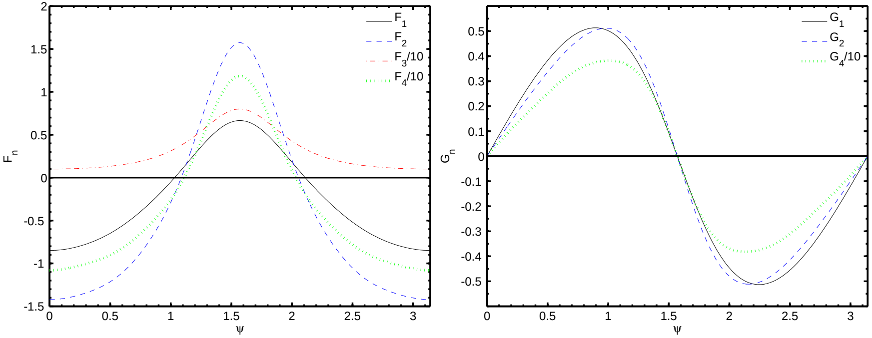
- What **is** the parameter space?

- How robust are these predictions, if we change the μ function?
Can we avoid rerunning *all* our numerical codes?
What if $\mu \to z^n$, $n \neq 1$?

- What if we change the relativistic modified gravity theory?

QN Angular Profile Functions

$$\mu = 1 - \frac{C_1^\mu}{z^p} + \cdots \Rightarrow \mathbf{U} = \frac{r}{r_0}\mathbf{N}(\psi) + \left(\frac{r_0}{r}\right)^{p-1}\mathbf{B}_p(\psi)$$



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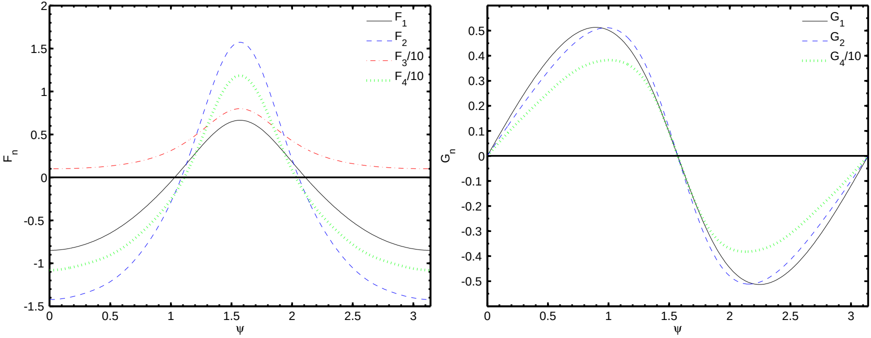
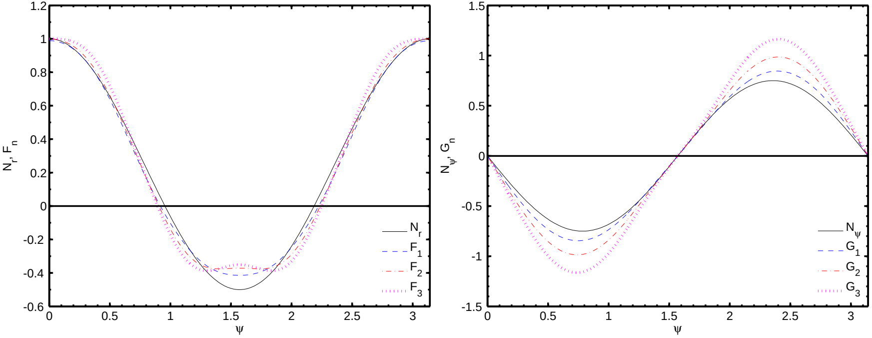


$$-\nabla\phi = \frac{4\pi a_0}{\kappa}\left[\frac{r}{r_0}\mathbf{N} + C_1^\mu\left(\frac{r}{r_0}\right)^{1-p}\left(\frac{\mathbf{N}}{N^p} + \frac{p}{2}\mathbf{B}_p^r\right) + \mathcal{O}(r^{1-2p}) + \ldots\right.$$

DM Angular Profile Functions

$$\mu = z^n + \cdots \Rightarrow \mathbf{U} = C(n,p)\left(\frac{r}{r_0}\right)^{\alpha(n)-2} \mathbf{D}_n(\psi)$$



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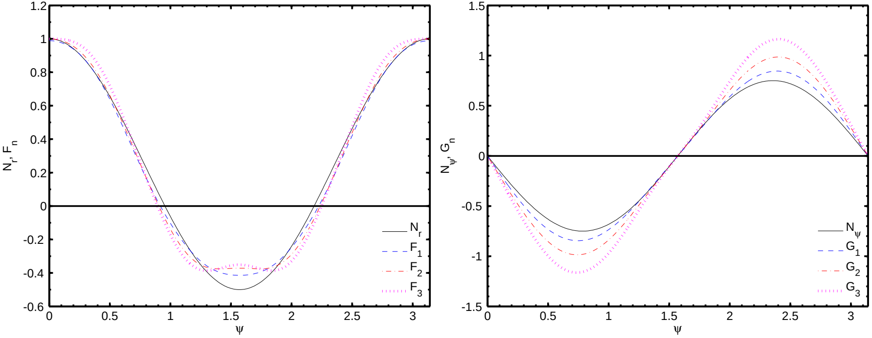
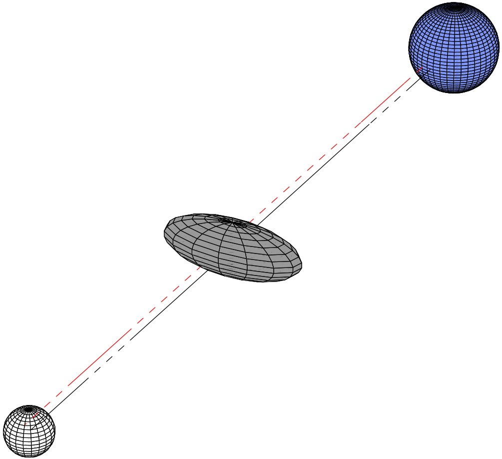


$$-\nabla\phi = \frac{4\pi a_0}{\kappa}\left[C(n,p)^{\frac{1}{n+1}}\left(\frac{r}{r_0}\right)^{\frac{\alpha(n)-2}{n+1}} \frac{\mathbf{D_n}}{D_n^{\frac{n}{n+1}}}\right]$$

Lunar Laser Ranging/Very-long-baseline interferometry

Additional possibility for testing MOND like theories by measuring the time delay from photons moving through the anomalous Earth-Moon saddle potential during a lunar eclipse is another route to constraining μ (or comparing emission from a quasar through the Jupiter-Sun saddle):



LLR/VLBI

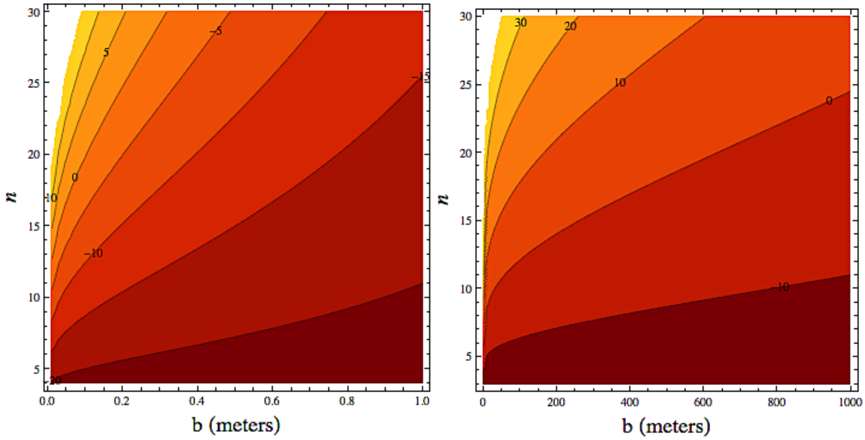
$$\mu \simeq \frac{1}{z^n}$$

LLR/VLBI

$$\mu \simeq \frac{1}{z^n} \Rightarrow \phi = -\frac{C_1}{r^{\alpha-2}}(f_0 + f_2 \cos 2\psi + \dots)$$

LLR/VLBI

$$\mu \simeq \frac{1}{z^n} \Rightarrow \phi = -\frac{C_1}{r^{\alpha-2}}(f_0 + f_2 \cos 2\psi + \dots) \Rightarrow \Delta t \approx \frac{2\pi f_0}{b} \frac{a_0^4}{c^3 A^3}$$



A Few More Caveats...

Type I, Physical potential $= \Phi = \Phi_N + \phi$ (eg TeVeS)

$$\nabla \cdot \left[\mu\left(\frac{\kappa}{4\pi}\frac{|\nabla\phi|}{a_0}\right)\nabla\phi\right] = \frac{\kappa}{4\pi}\nabla^2\Phi_N \qquad \mu(z) \to z, z \ll 1$$

Type II, Physical potential $= \Phi = \Phi_N + \phi$ (eg BiMOND)

$$\nabla^2\phi = \frac{\kappa}{4\pi}\nabla \cdot \left[\nu\left(\left(\frac{\kappa}{4\pi}\right)^2\frac{|\nabla\Phi_N|}{a_0}\right)\nabla\Phi_N\right] \qquad \nu(w) \to \frac{1}{\sqrt{w}}, w \ll 1$$

Type III, Physical potential $= \Phi$ (eg Einstein Æther)

$$\nabla \cdot \left[\tilde{\mu}\left(\frac{|\nabla\Phi|}{a_0}\right)\nabla\Phi\right] = \nabla^2\Phi_N \qquad \tilde{\mu}(y) \to y, y \ll 1$$

How robust are our predictions in each case? **(Work in progress)**

Conclusions

Whilst we have the main ingredients, ie our gravitational theories and cosmological models, in order to understand the early universe, it is prudent to consider alternatives in areas which are still not well understood, eg the dark sector - **the case for testing and constraining modified gravity with LPF, cleanly and without 'messy' astrophysics remains an exciting possibility...**

Further work needs to be done to understand how these results vary with different choices of free function, compute SNRs and complementing these effects with LLR/VLBI experiments across the saddle, detections of chameleon mechanisms, non-gaussianities... **In Progress**

There exists a plethora of relativistic theories (and their cosmologies) and although there are essentially only *3 different NR limits*, untangling them is ongoing, developing a SP formalism would be an ideal goal... **(PPS?)**