Numerically calculating inflationary correlation functions

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e.g. arXiv:1302.3842, DJM
arXiv:1008.3159, DJM, David Seery, Daniel Wesley
on going work with David Seery, Mafalda Dias, Joe Elliston, Jonny Frazer,
arXiv:1401.6078 with John Ellis and Nick Mavromatos

Things I'll say something about

- What are inflationary correlation functions?
- Approaches to calculating them
- When are numerics required?
- Our implementation (work in progress)
- Some results

Basics

- We care about the Fourier space correlation functions:

$$\langle \zeta(k_1)\zeta(k_2)\rangle = (2\pi)^3 P(k_1)\delta^3(\mathbf{k_1}+\mathbf{k_2})$$

$$\langle \zeta(k_1)\zeta(k_2)\zeta(k_3)\rangle = (2\pi)^3 B(k_1,k_2,k_3)\delta^3(\mathbf{k_1}+\mathbf{k_2}+\mathbf{k_3})$$

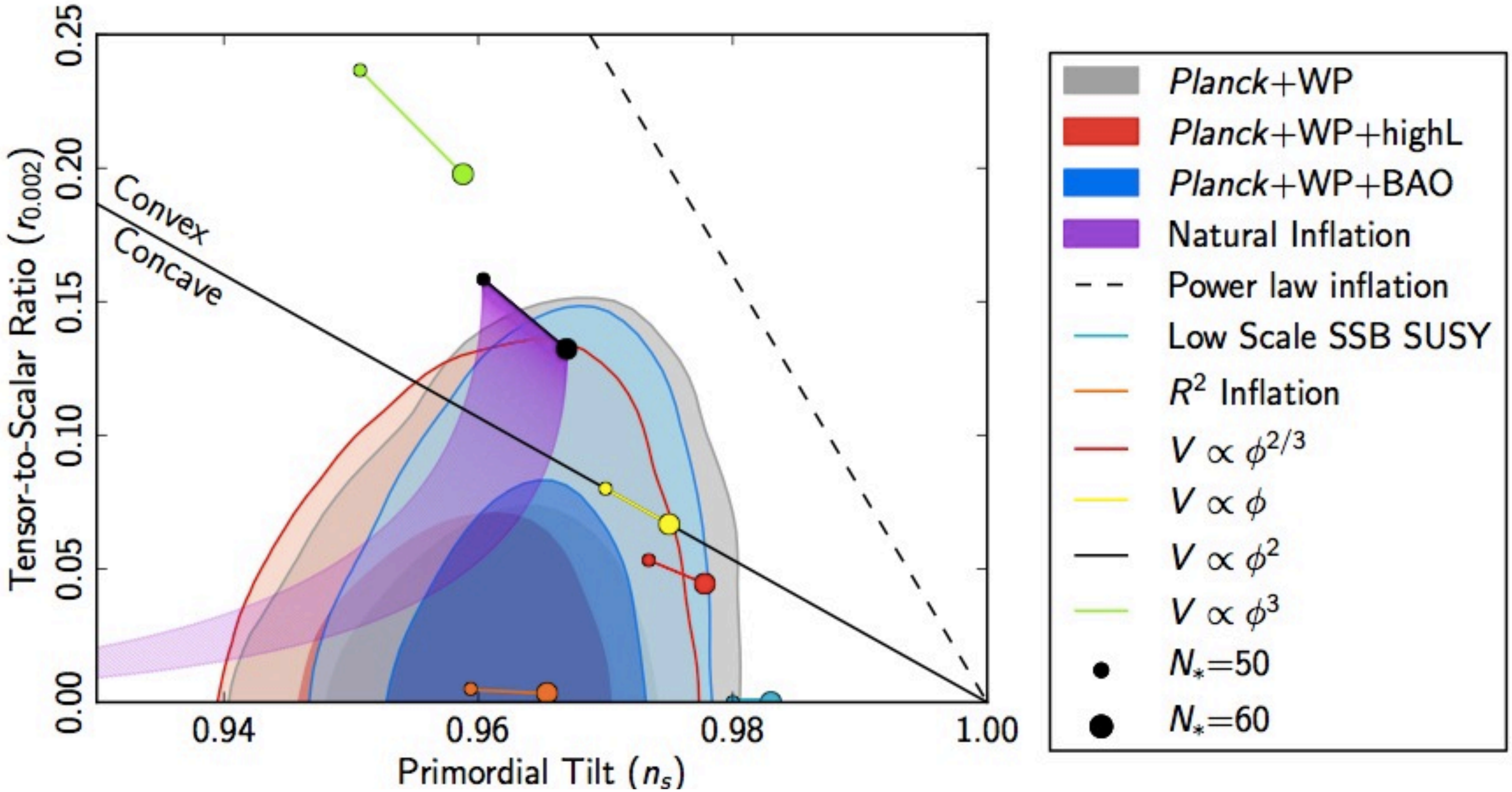
- Where for inflation

$$P(k) \approx Ak^{-3}$$

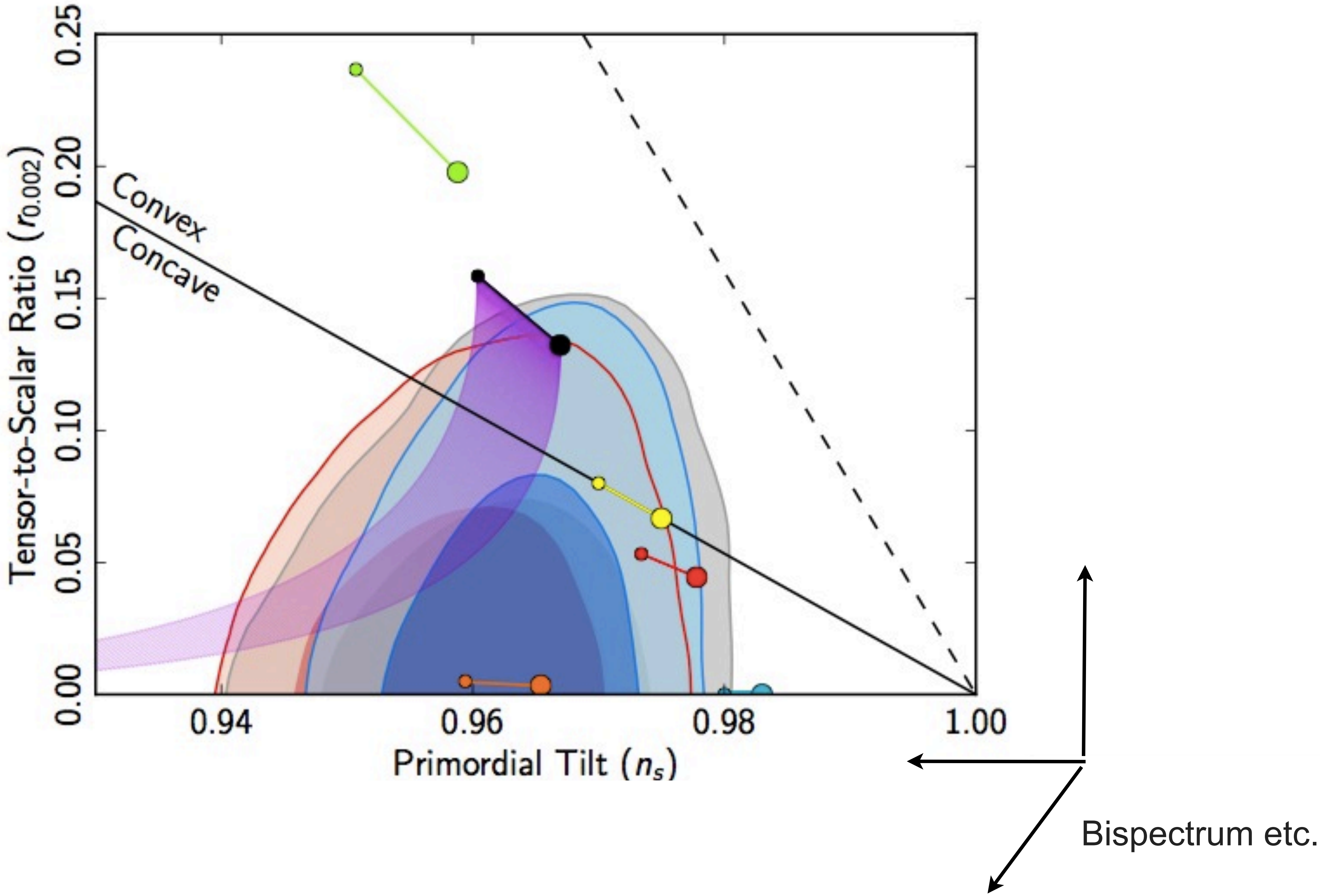
- And for vanilla inflation

$$f_{\rm nl}(k_1,k_2,k_3) \sim \text{slow roll parameters}$$

- Cosmological perturbation theory (e.g. review of Malik and Wands 2008), provides evolution equations/Lagrangian of perturbations (curvature/isocuvature or fields).



The Planck team - Ade et al. 2013



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Evolution of perturbations

- The equations of motion for fluctuations

$$x_{\alpha'} = \{\delta\phi_{a'}, \delta\dot{\phi}_{b'}\}$$

- are of form

$$\frac{\mathrm{d}x_{\alpha'}}{\mathrm{dt}} = u_{\alpha'\beta'}x_{\beta'} + \frac{1}{2!}u_{\alpha'\beta'\gamma'}\Big(x_{\beta'}x_{\gamma'} - \langle x_{\beta'}x_{\gamma'}\rangle\Big) + \ldots$$

- Where for example

$$\frac{\mathrm{d}\delta\phi_a(k)}{dt} = \delta\dot{\phi}_a(k)$$

$$\frac{\mathrm{d}\delta\dot{\phi}_a(k)}{\mathrm{d}t} = -3H\delta\dot{\phi}_a(k) - \frac{k^2}{a^2}\delta\phi_a(k) - M^2_{ab}\delta\phi_b(k) - V'''_{abc}[\delta\phi_b * \delta\phi_c](k) + \ldots$$

Usual approach

- The fields must be solved as operators from some initial vacuum.

- Traditional QFT 'In-In' approach is to start with linear field

$$\delta\phi_a(t,\mathbf{k}) = \Psi_{ac}(t,k)a_c(\mathbf{k}) + \Psi^*_{ac}(t,k)a^\dagger_c(-\mathbf{k})$$

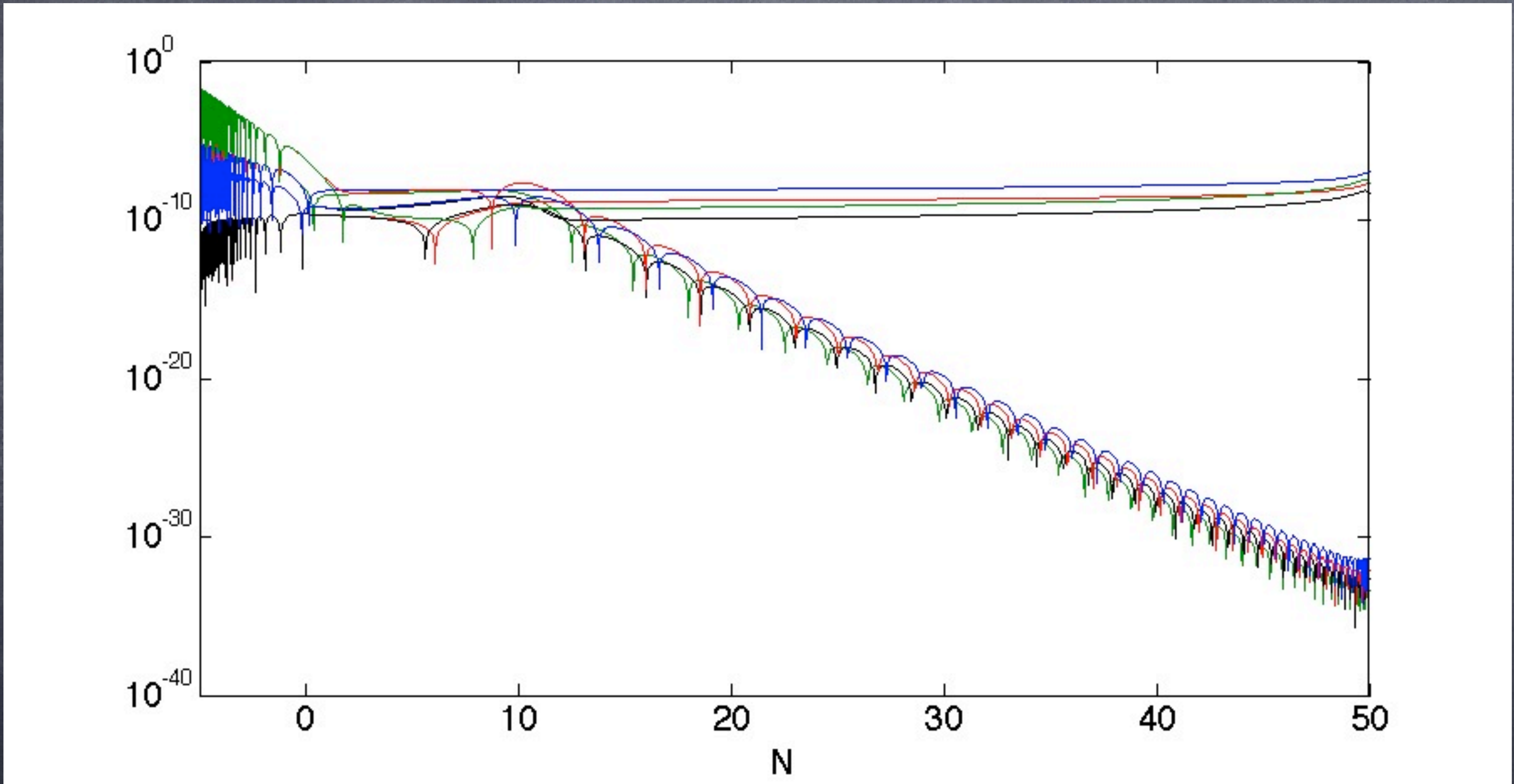
- Solve for the coupling matrix (Salopek, Bond and Bardeen, 1989, Huston, Pyflation, 2012, Easther et al. 2013) to find two point,

$$\langle \hat{\delta\phi}_a \hat{\delta\phi}_b \rangle = \Psi_{ac}\Psi^*_{cb}$$

- and use (Maldacena, 2003)

$$\langle \hat{x}_{\alpha'}\hat{x}_{\beta'}\hat{x}_{\gamma'} \rangle = -i\int_{t_0}^{t} \mathrm{d}t' \langle \left[\hat{x}_{\alpha'}\hat{x}_{\beta'}\hat{x}_{\gamma'}, \hat{\mathcal{H}}_{\rm int}(t') \right] \rangle$$

- Example $\{\Psi_{ab}, \dot{\Psi}_{ab}\}$ for double quadratic



Some comments

- Single field versus multiple field: initial conditions, evolution after horizon crossing, gauge issues, curved field space metric, reheating..... makes things harder, particularly for bispectrum

- Analytical calculations are limited (particularly for multi-fields, δN)

- Previous work for numerical calculation of bispectrum -- full calculation for single field (Chen, Easther, Lim, 2006, 2008, Horner and Contaldi, 2013), super-horizon using δN (e.g. Elliston, Mulryne, Seery, Tavakol, 2011, Leung, Tarrant, Byrnes, Copeland, 2012), using transport (Mulryne, Seery, Wesley, 2010, Dias, Frazer, Liddle 2012)

- Interesting questions about how In-In, transport, δN related (Seery, Mulryne, Dias, Ribeiro, 2012, Mulryne 2013)

Transport derivation

- We have developed an alternative, 'transport' approach, (e.g. Mulryne, Seery, Wesley, 2009, Mulryne 2013), which directly solves for correlation functions:

$$\Sigma_{\alpha'\beta'} = \langle x_{\alpha'} x_{\beta'} \rangle \ , \ \alpha_{\alpha'\beta'\gamma'} = \langle x_{\alpha'} x_{\beta'} x_{\gamma'} \rangle$$

- Where

$$\Sigma_{\alpha'\beta'} = (2\pi)^3 \delta(\mathbf{k}_\alpha + \mathbf{k}_\beta) \Sigma_{\alpha\beta}(k_\alpha)$$

$$\alpha_{\alpha'\beta'\gamma'} = (2\pi)^3 \delta(\mathbf{k}_\alpha + \mathbf{k}_\beta + \mathbf{k}_\gamma) \alpha_{\alpha\beta\gamma}(k_\alpha, k_\beta, k_\gamma)$$

- And using Ehrenfest's theorem, we find

$$\frac{\Sigma^r_{\alpha\beta}(k_\alpha)}{\mathrm{d}t} = u_{\alpha\gamma}(k_\alpha)\Sigma^r_{\gamma\beta}(k_\alpha) + u_{\beta\gamma}(k_\alpha)\Sigma^r_{\gamma\alpha}(k_\alpha)$$

$$\frac{\mathrm{d}\alpha_{\alpha\beta\gamma}(k_\alpha, k_\beta, k_\gamma)}{\mathrm{d}t} = u_{\alpha\lambda}(k_\alpha)\alpha_{\lambda\beta\gamma}(k_\alpha, k_\beta, k_\gamma) + u_{\alpha\lambda\mu}(k_\alpha, k_\beta, k_\gamma)\Sigma^r_{\lambda\beta}(k_\beta)\Sigma^r_{\mu\gamma}(k_\gamma) - \frac{1}{3}u_{\alpha\lambda\mu}(k_\alpha, k_\beta, k_\gamma)\Sigma^i_{\lambda\beta}(k_\beta)\Sigma^i_{\mu\gamma}(k_\gamma) + \mathrm{cyclic}$$

Transport numerical algorithm

- Step 1. Derive the u coefficients for the model at hand (multi-field canonical/non-canonical, curved field space etc).
- Step 2. Calculate the initial conditions (Bunch-Davis) - integral solutions can be used to fix these at arbitrary times (at or long before horizon crossing).
- Step 3. Solve the ODEs for the correlations of the field perturbations. If want the bi-spectra for example, one evolution for each triangle of k scales -- MPI, GPUs
- Step 4. Convert to any other quantity of interest (zeta correlations - power/bi-spectra - fnl.....)
- Step 5. Integrate bi-spectrum against template (local etc)
- We are developing user friendly code to release publicly

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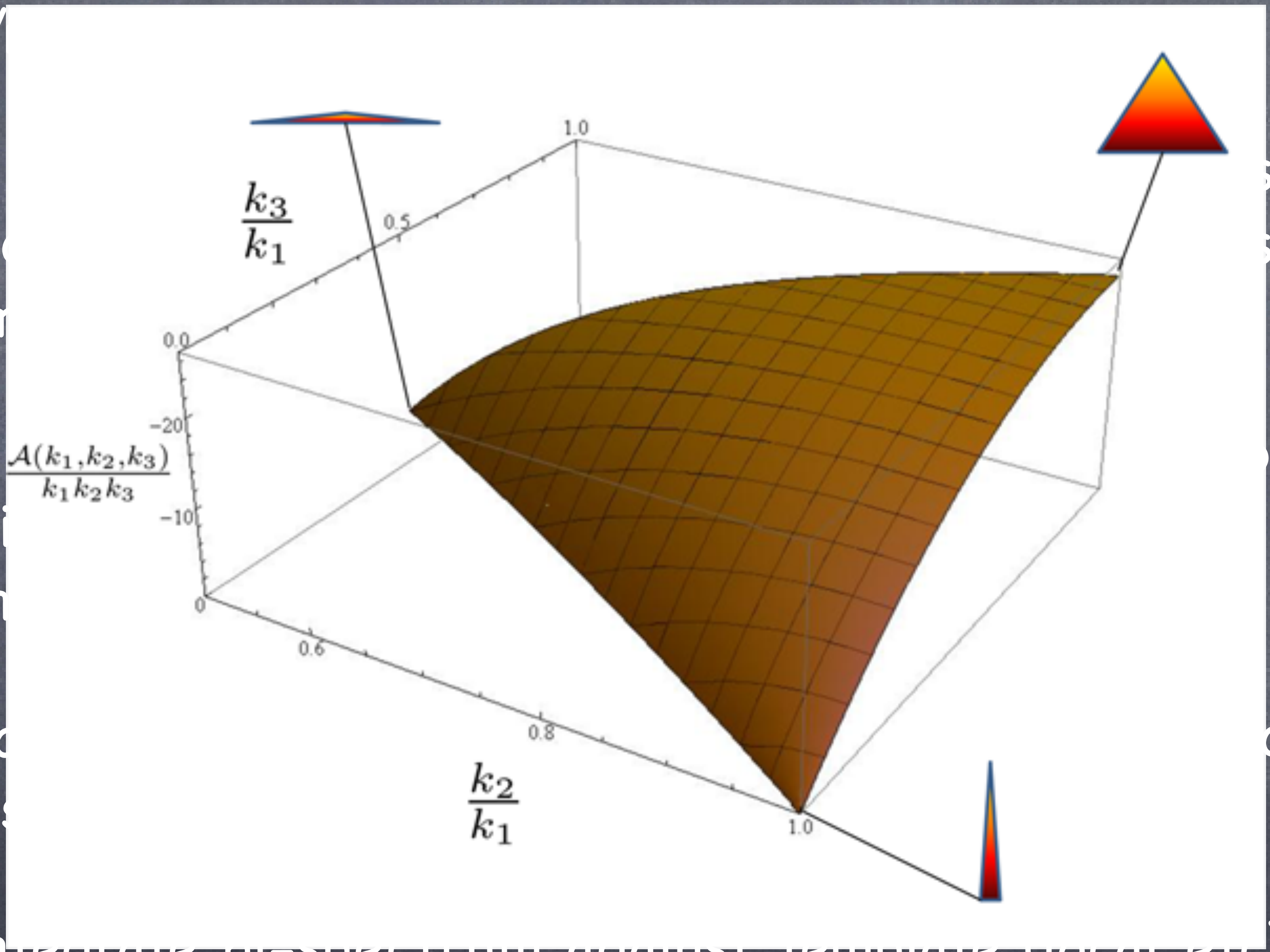

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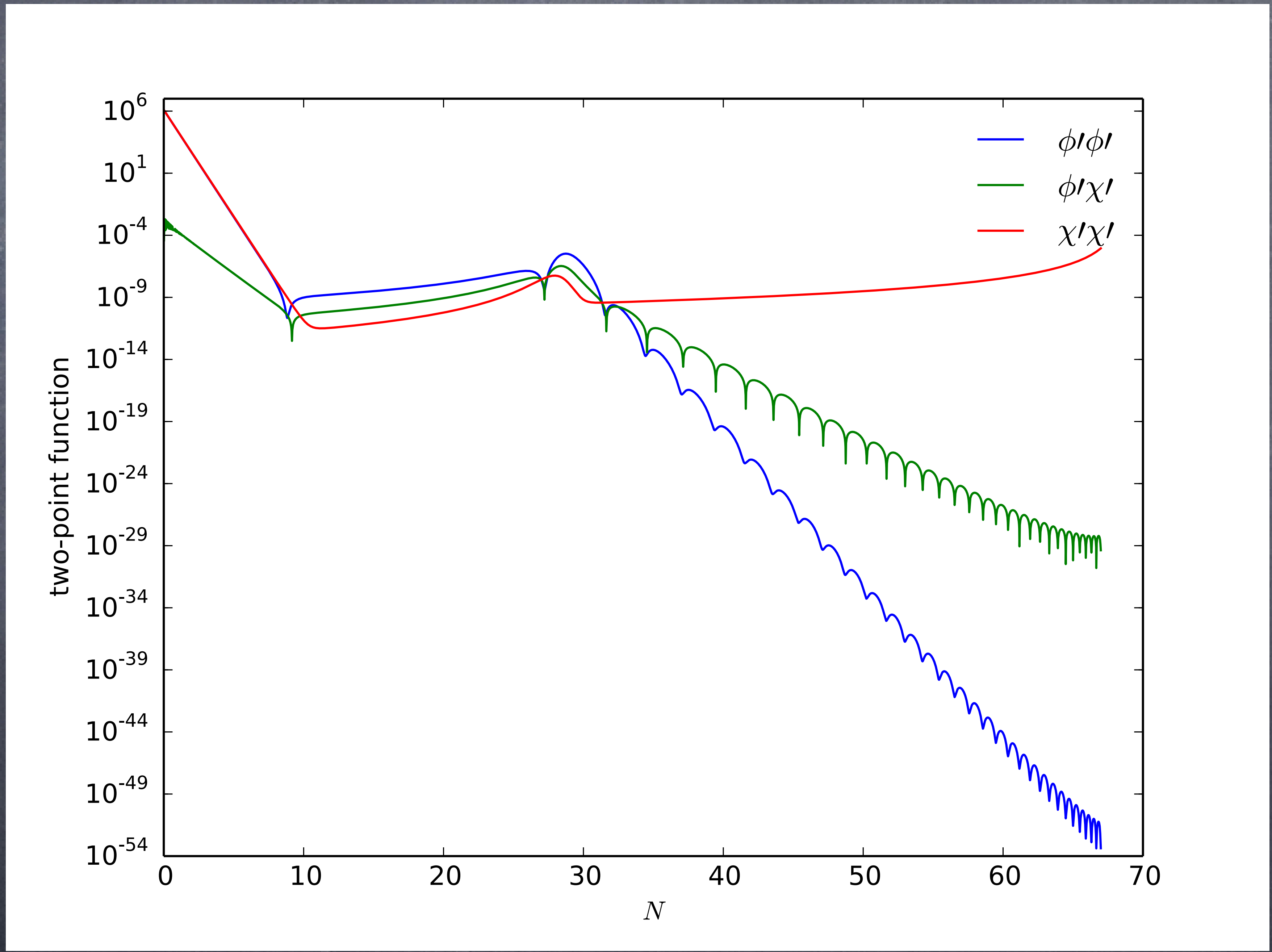

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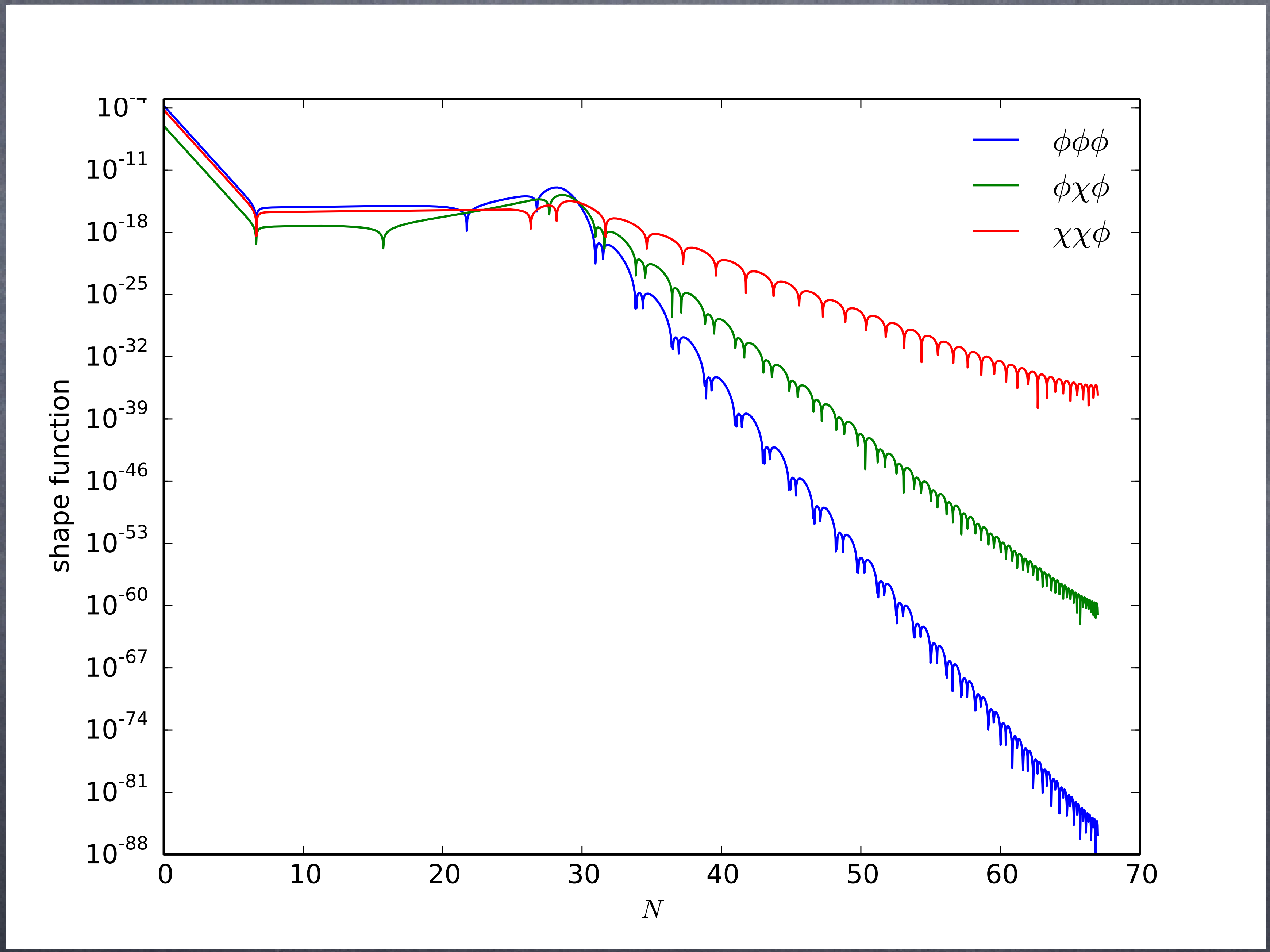

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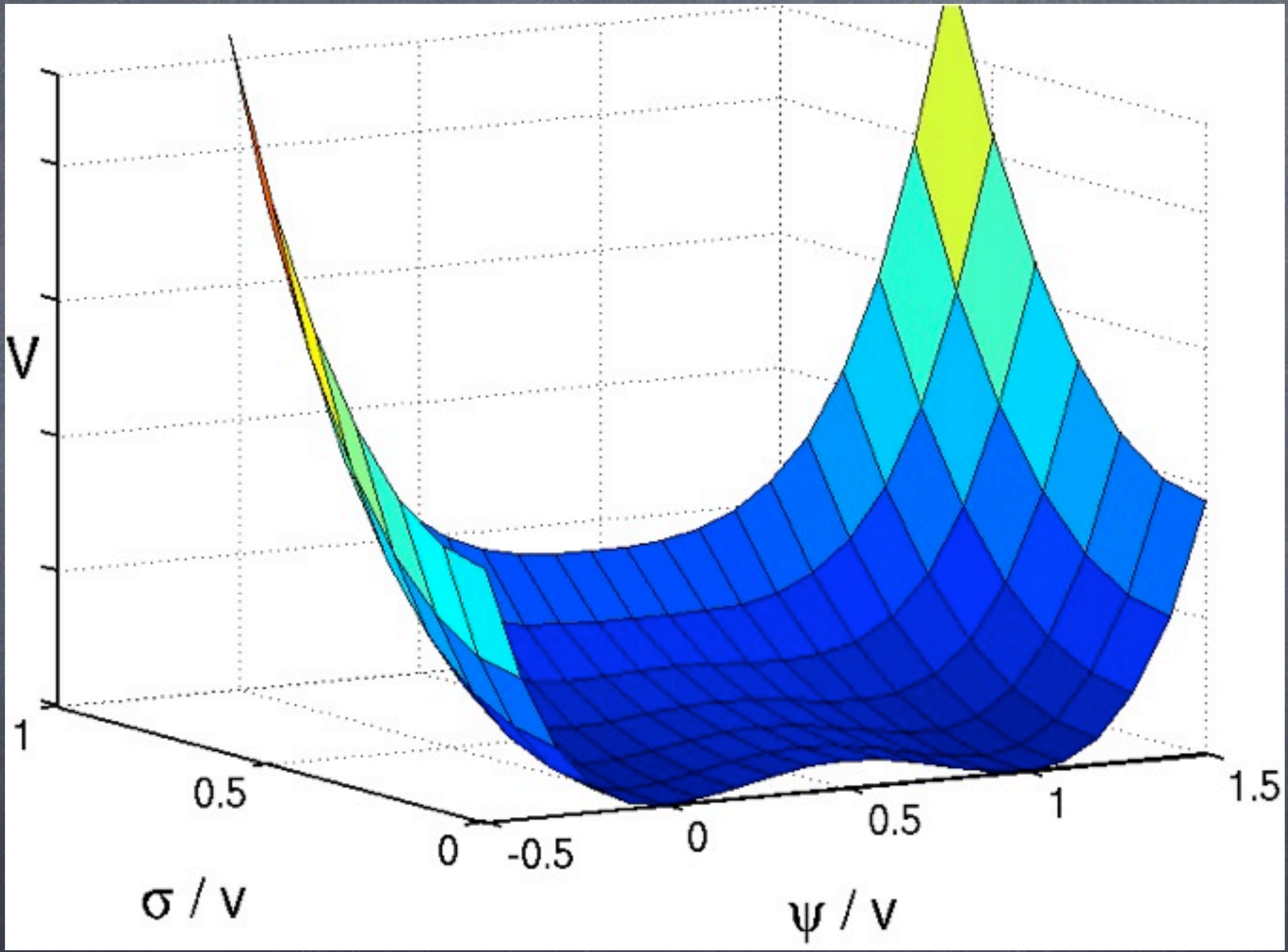

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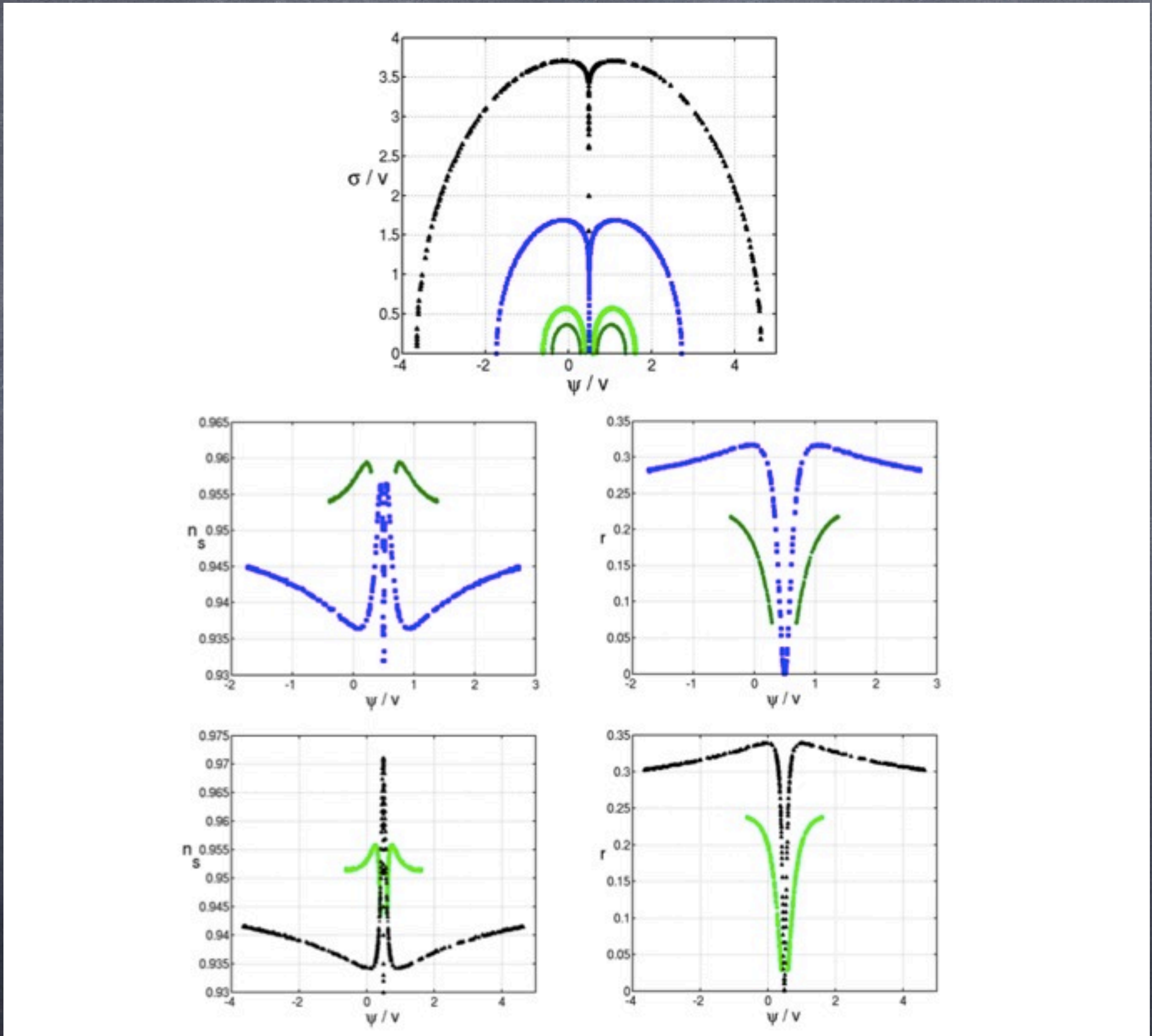

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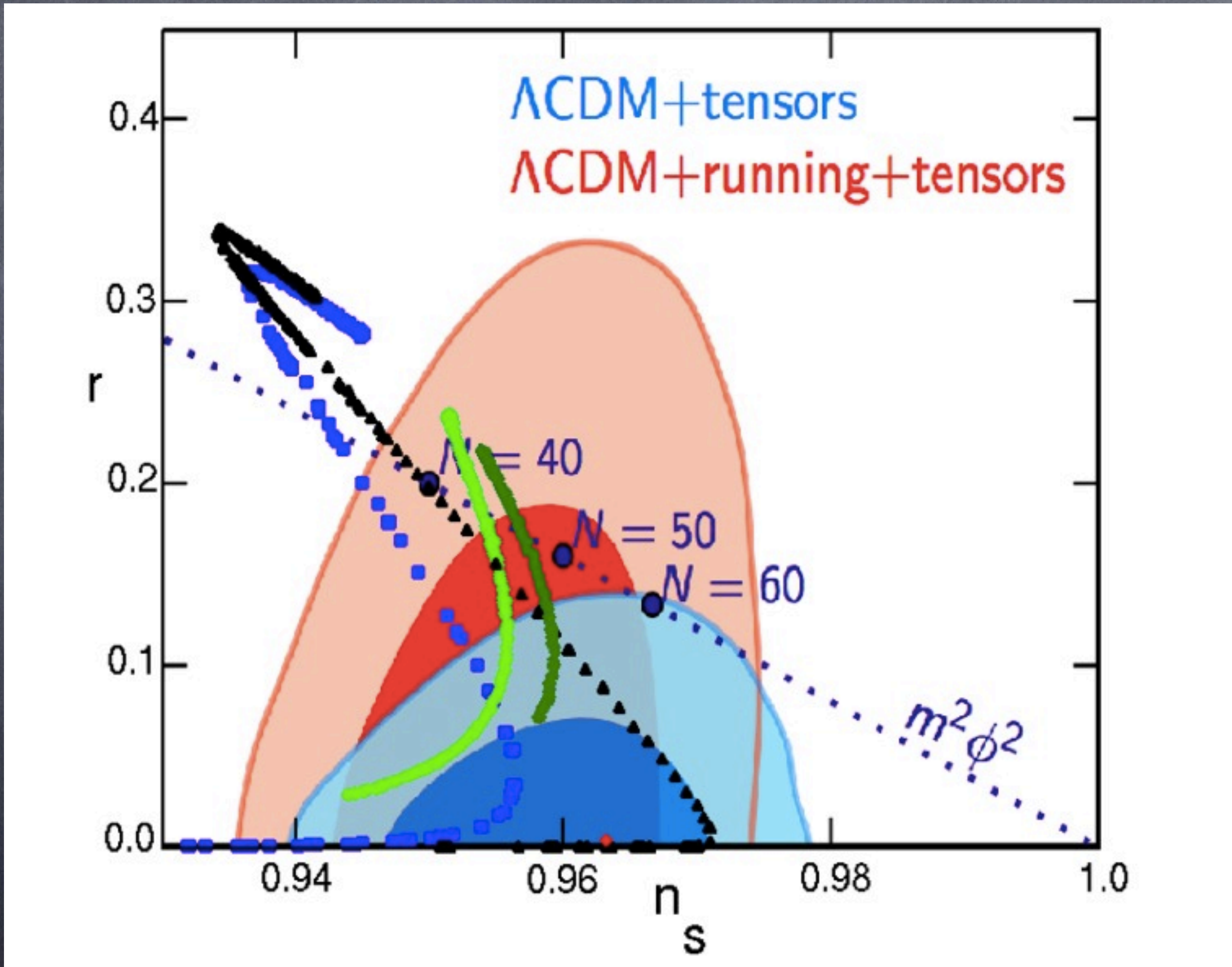

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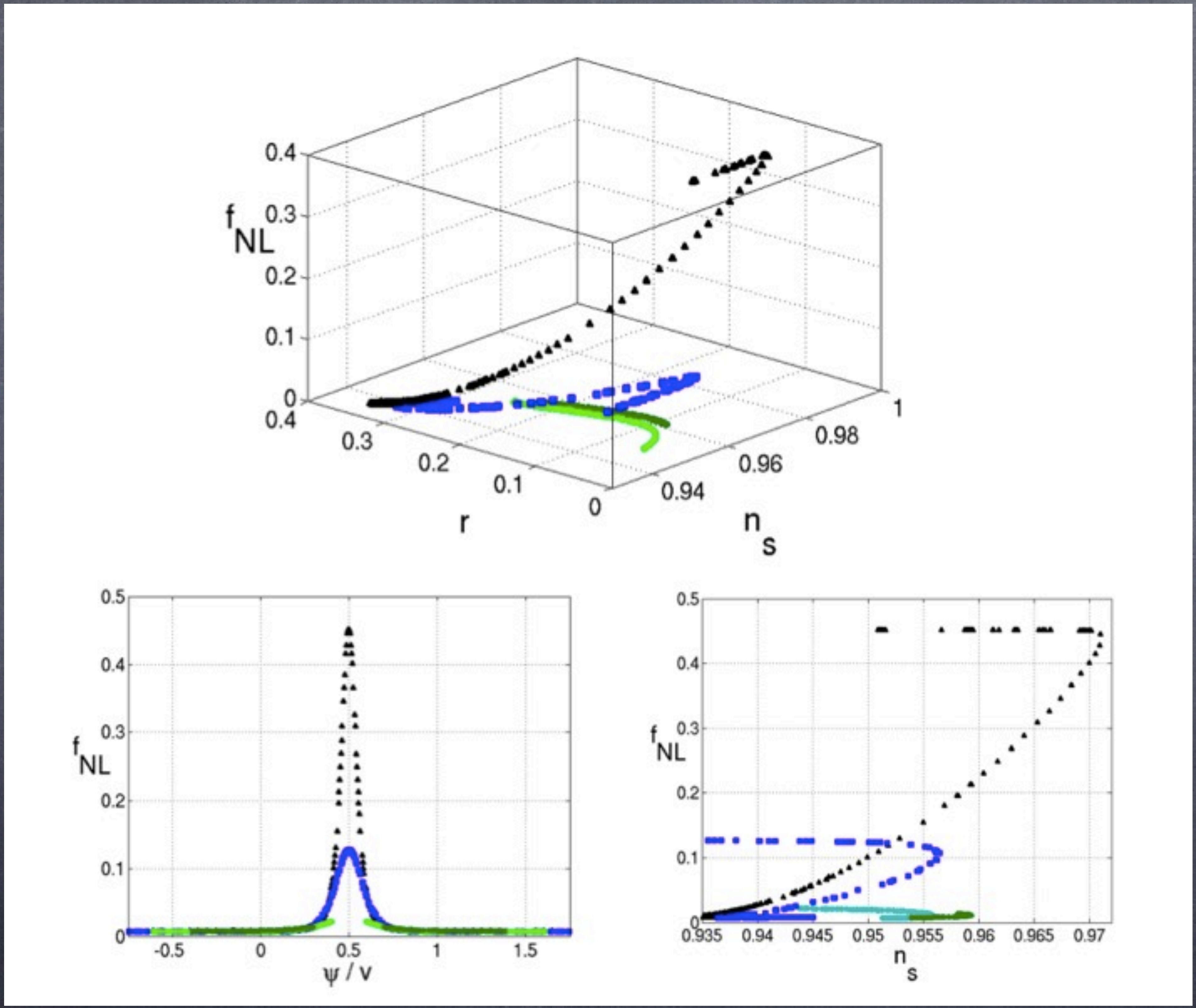

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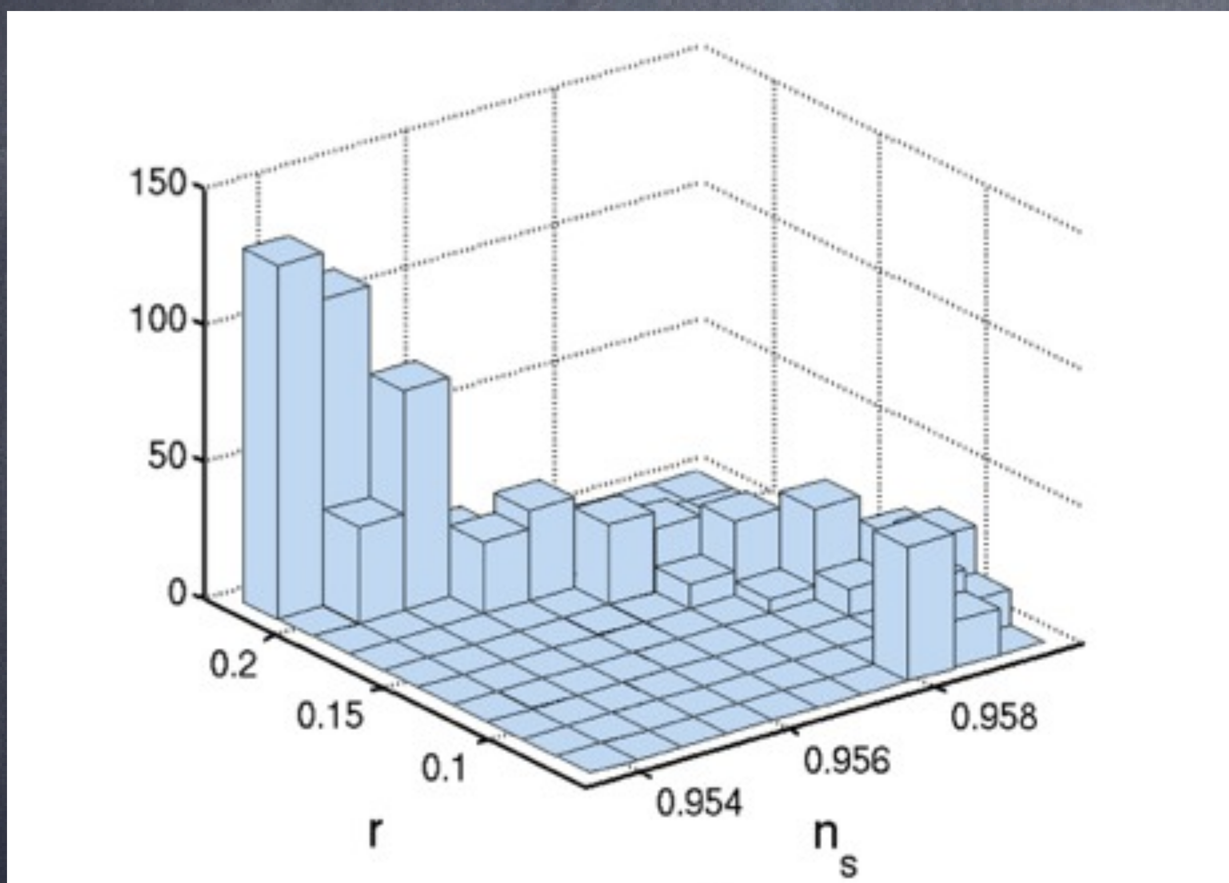

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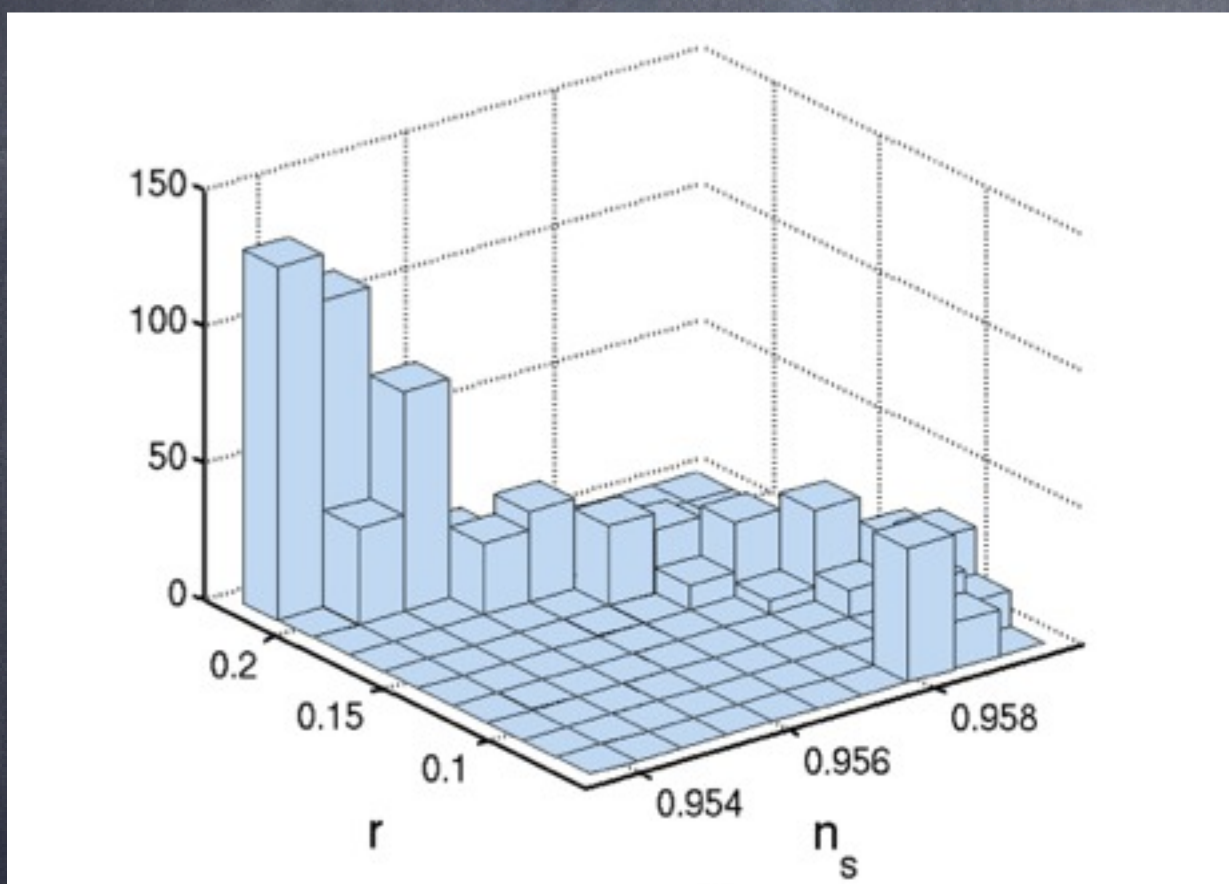

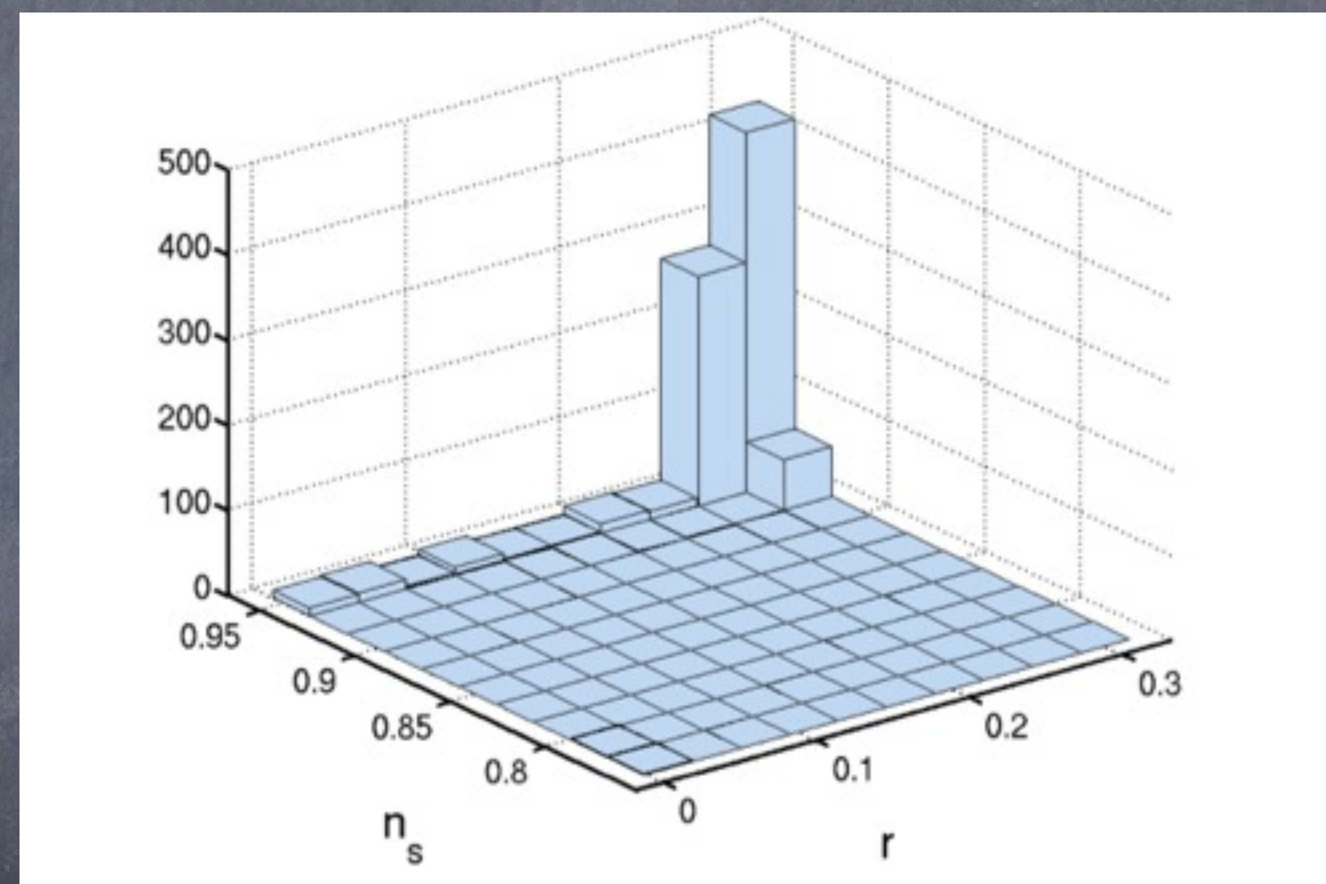

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