# Soft limits in multi-field inflation

David J. Mulryne

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based on arXiv:1507.08629 and forthcoming work with Zac Kenton

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THE ROYAL SOCIETY

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# Intro to soft limits

- Observations constrain correlations
- Soft limits occur when we consider a correlation with a large hierarchy of scales
- Often simple to calculate **theoretically**, and can lead to beautiful relations between correlations
- Important for comparison with **observation**, as range of scales probed increases, and for specific observations (halo bias)

# Intro to soft limits

- Simplest example (Maldacena '03; Creminelli, Zaldarriaga '04; Cheung et al. '08)
  - Power spectrum $\langle \zeta_{\vec{k}} \zeta_{\vec{k'}} \rangle = (2\pi)^3 \delta(\vec{k} + \vec{k'}) P_\zeta(k)$
  - Bispectrum $\langle \zeta_{\vec{k_1}} \zeta_{\vec{k_2}} \zeta_{\vec{k_3}} \rangle = (2\pi)^3 \delta(\vec{k_1} + \vec{k_2} + \vec{k_1}) B_\zeta(k_1, k_2, k_3)$

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$$\lim_{k_1 \ll k_3} B_\zeta(k_1, k_2, k_3) = -\frac{d \log(k_3^3 P_\zeta(k_3))}{d \log k_3} P_\zeta(k_1) P_\zeta(k_3)$$
$$= -(n_s - 1) P_\zeta(k_1) P_\zeta(k_3)$$

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  - Trispectrum $\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \zeta_{\vec{k}_4} \rangle = (2\pi)^3 \delta(\vec{k}_1 + \vec{k}_2 + \vec{k}_3 + \vec{k}_4) T_{\zeta}(k_1, k_2, k_3, k_4)$

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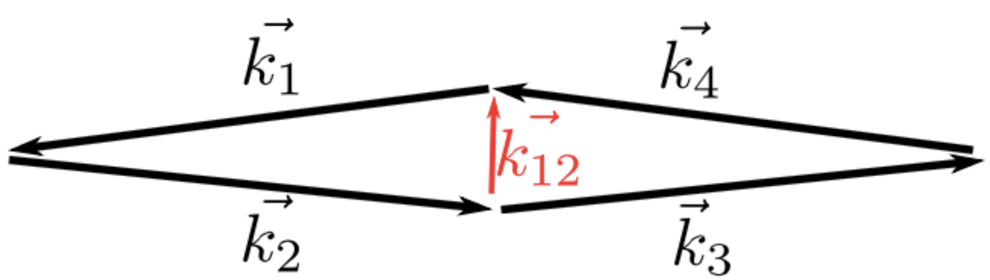
$$\tilde{f}_{\rm nl} \equiv \lim_{k_1 \to 0} \frac{B_\zeta(k_1, k_2, k_3)}{P_\zeta(k_1) P_\zeta(k_2)} \,, \quad \tilde{\tau}_{\rm nl} \equiv \lim_{|\vec{k}_1 + \vec{k}_2| \to 0} \frac{T_\zeta(k_1, k_2, k_3, k_2)}{P_\zeta(k_1) P_\zeta(k_3) P_\zeta(|\vec{k}_1 + \vec{k}_2|)}$$

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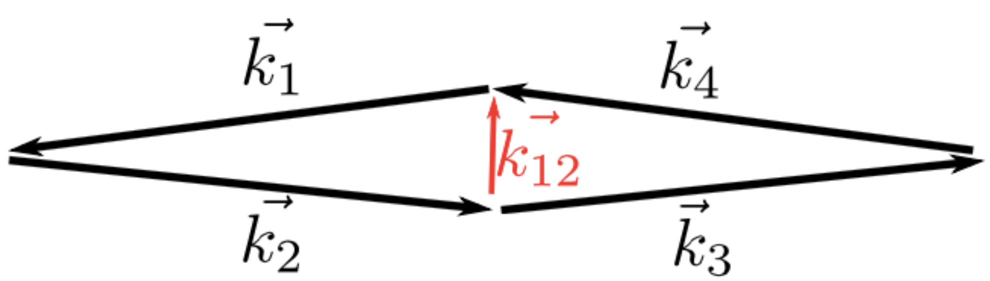


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- Extensions to higher point functions and multiple soft limits, other fields (e.g. Mirbabayi and Zaldarriaga et al. '14; Joyce et al. '14)

# Calculating observables with multiple fields

- Want to track correlations of the fluctuations, $\delta\phi$ etc, ultimately want curvature perturbation, $\zeta$ , power-spectrum, bispectrum, trispectrum etc.

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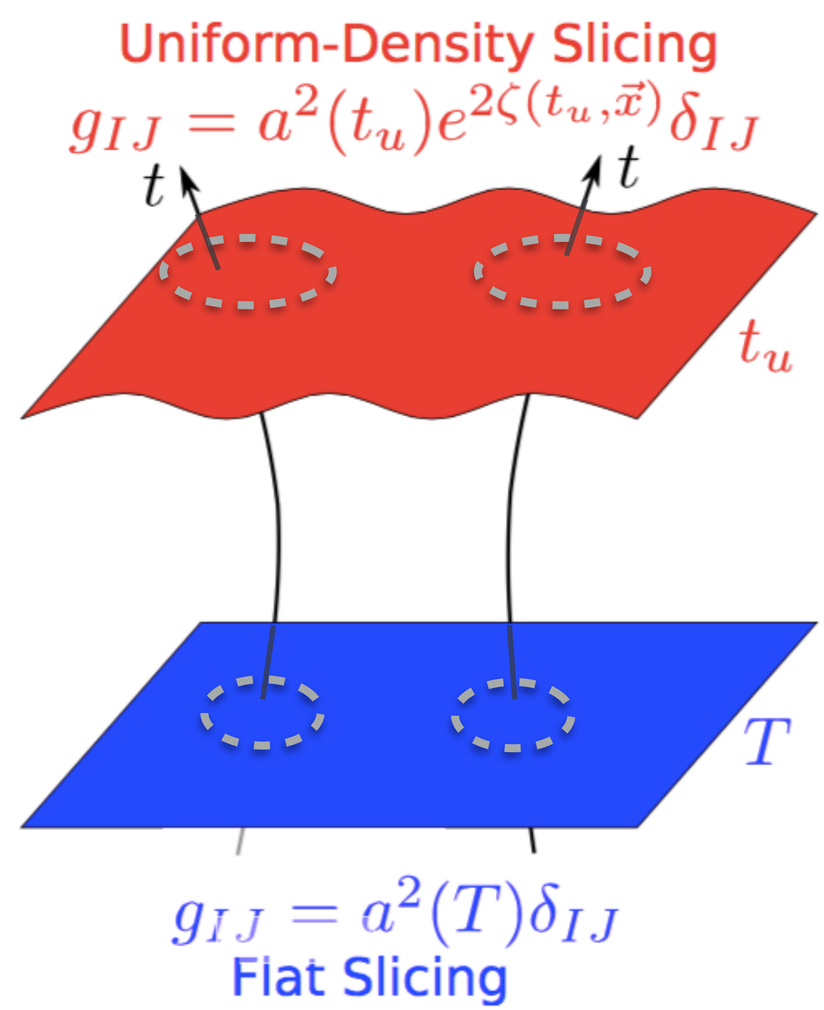
$$\zeta(t_u,\vec{x}) = N_i^{(T)}\delta\phi_i^{(T)}(\vec{x}) + \frac{1}{2}N_{ij}^{(T)}\delta\phi_i^{(T)}(\vec{x})\delta\phi_j^{(T)}(\vec{x}) + \ldots$$

where $N_i^{(T)} \equiv \frac{\partial N_0(t_u,T)}{\partial\phi_i^{(T)}}$ with $N_0(t_u,T) \equiv \int_T^{t_u} H(t)dt$




(e.g. Wands et al., ’00)

# Calculating observables with multiple fields

- In-In calculations can give us correlations at horizon crossing for $k_1 \sim k_2 \sim k_3$

$$\langle \delta\phi^{(T)}_{i,\vec{k_1}} \delta\phi^{(T)}_{j,\vec{k_2}} \rangle = (2\pi)^3 \delta(\vec{k_1} + \vec{k_2}) \Sigma^{(T)}_{ij}(k_1)$$

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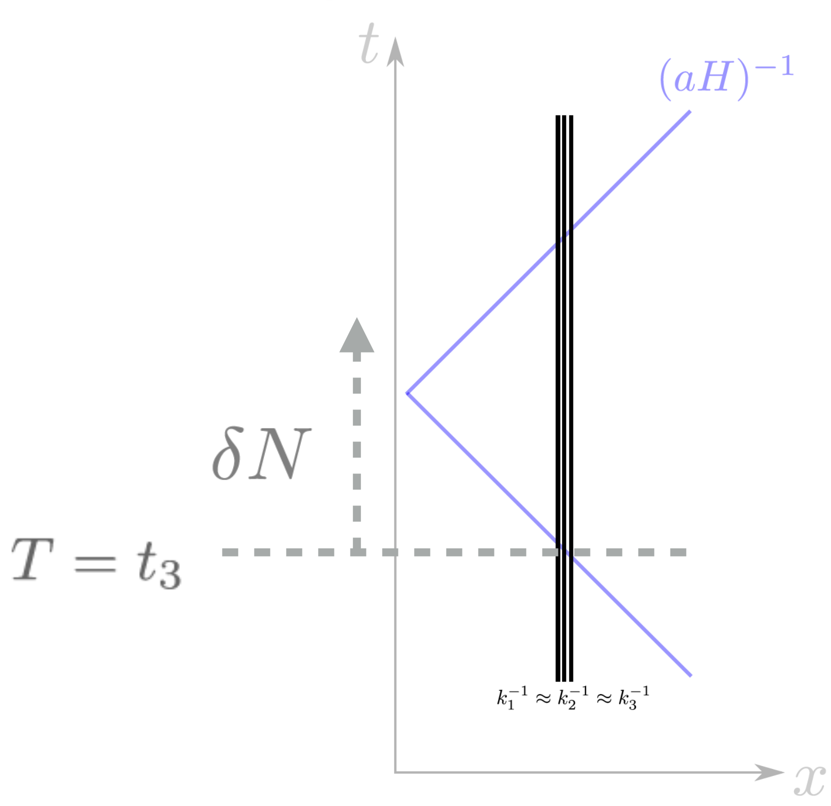
**Exit Times: Equilateral**



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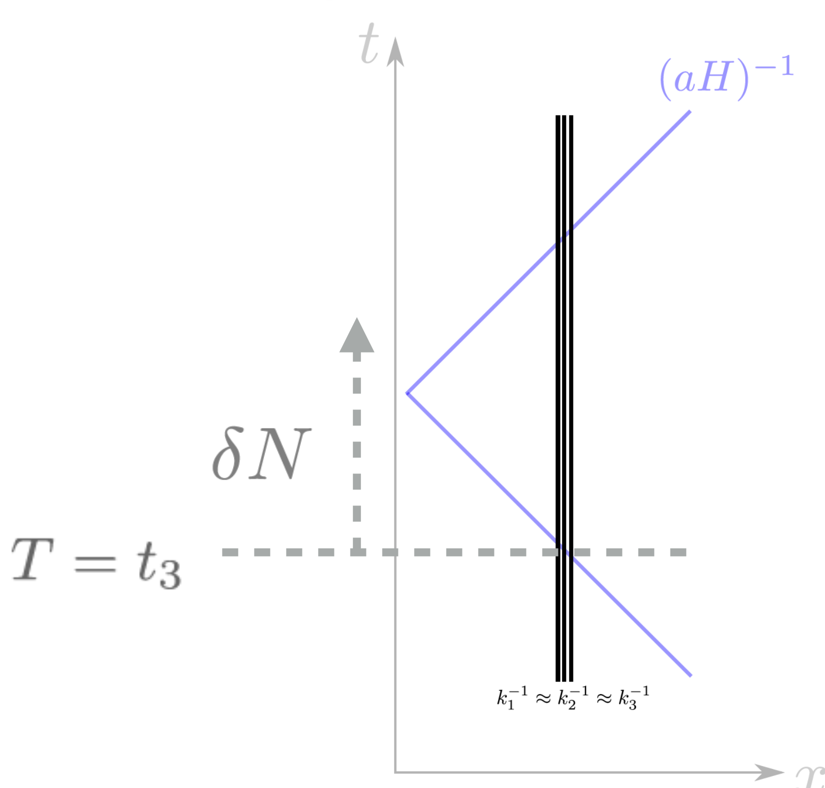


$$B_\zeta(k_1, k_2, k_3) = N^{(T)}_i N^{(T)}_j N^{(T)}_k \alpha^{(T)}_{ijk}(k_1, k_2, k_3) + N^{(T)}_i N^{(T)}_{jk} N^{(T)}_l \left[ \Sigma^{(T)}_{ij}(k_1) \Sigma^{(T)}_{kl}(k_2) + 2 \text{ perms} \right]$$

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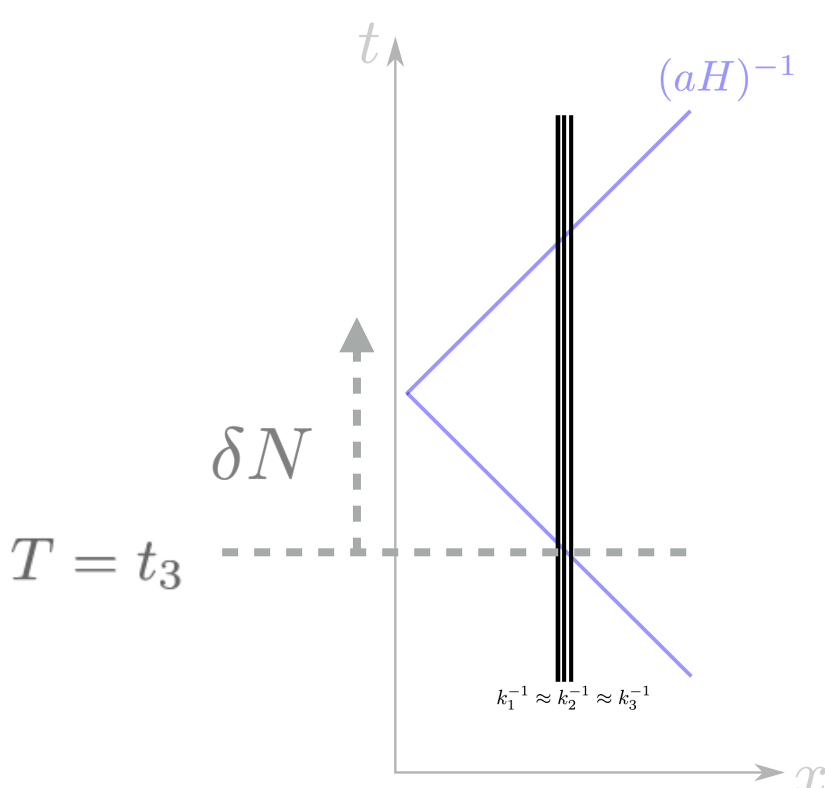


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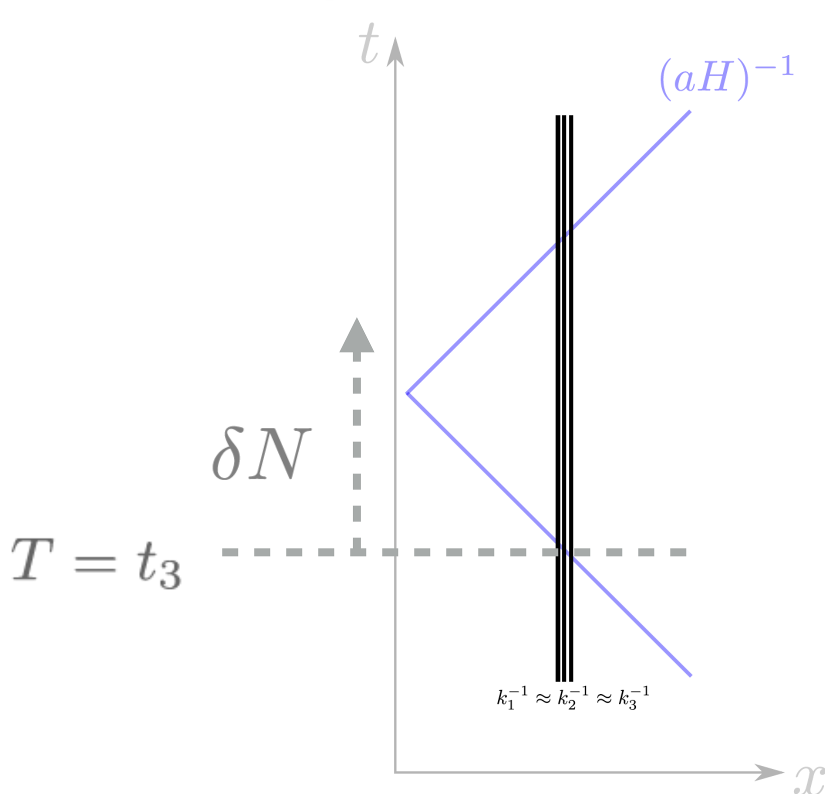


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(Seery and Lidsey '05; Lyth '05)

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- Analytic calculations difficult for correlations at times after horizon crossing (c.f. results of Byrnes et al. '09; Dias et al. '12)

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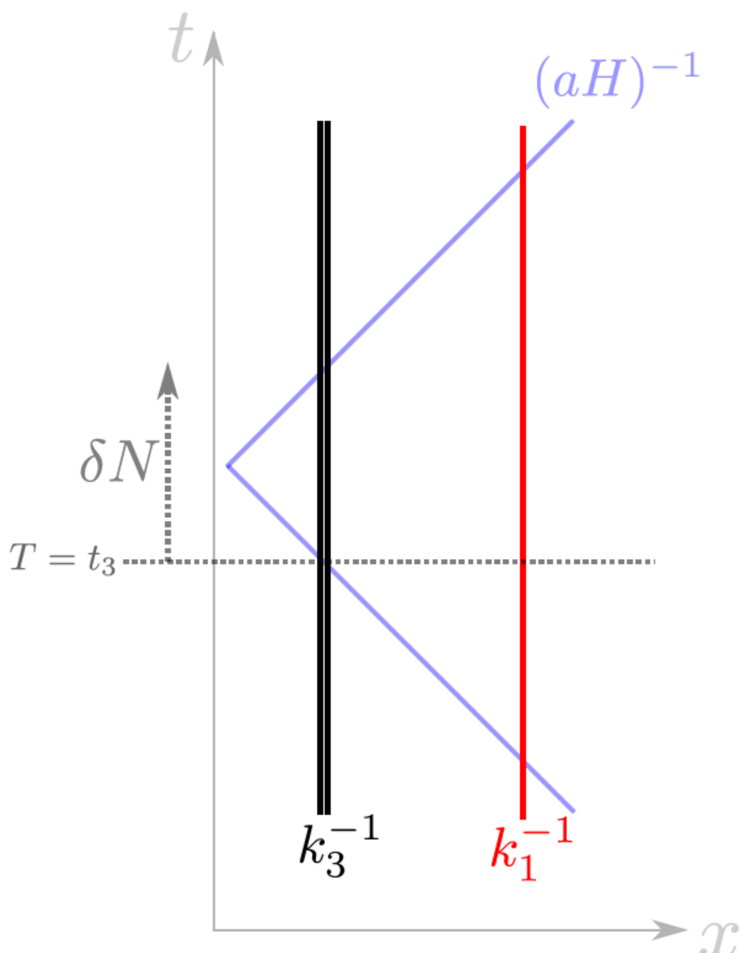

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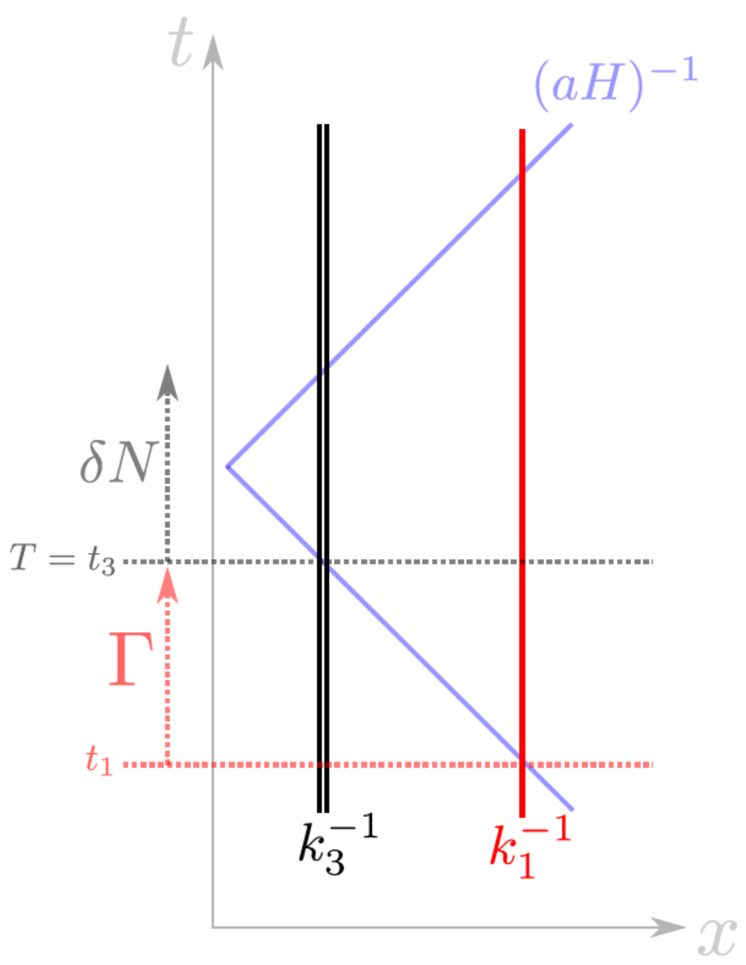

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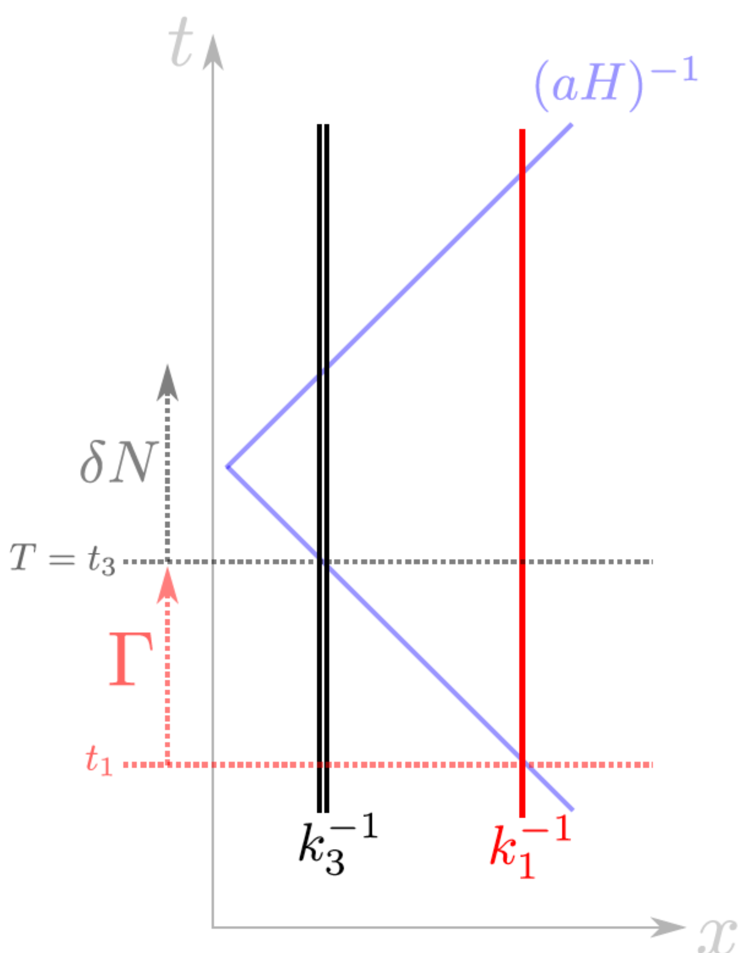


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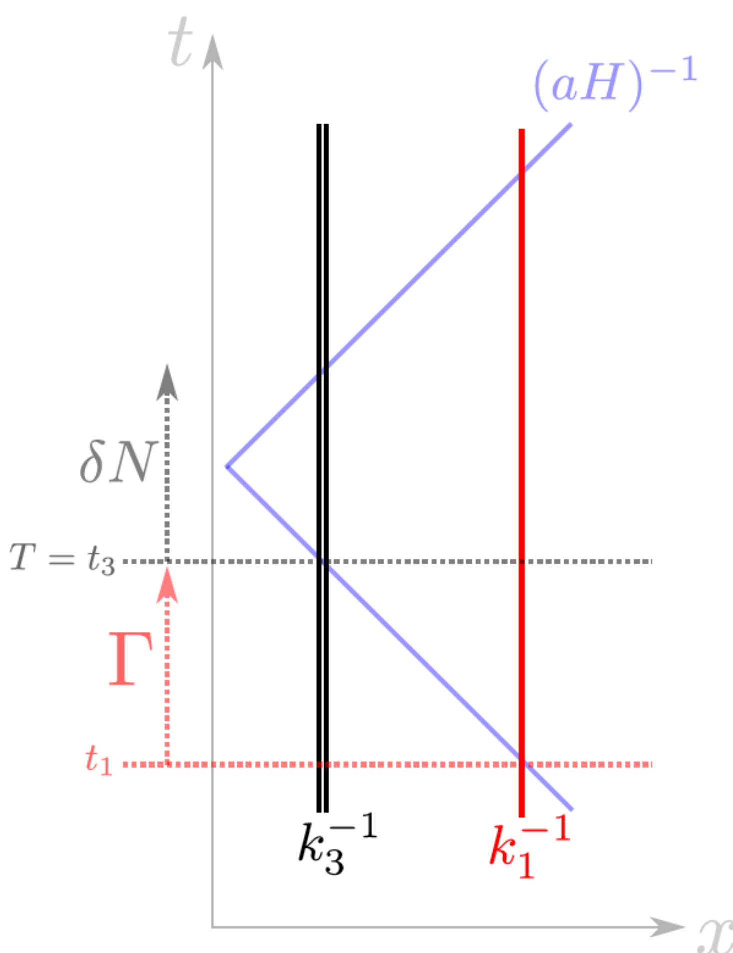


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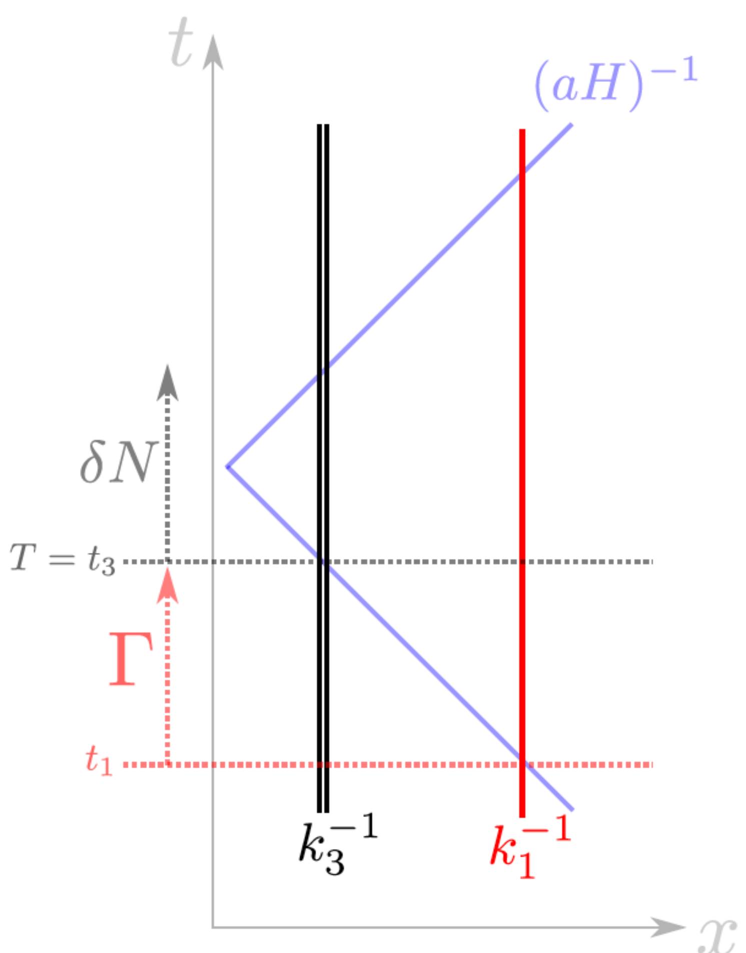


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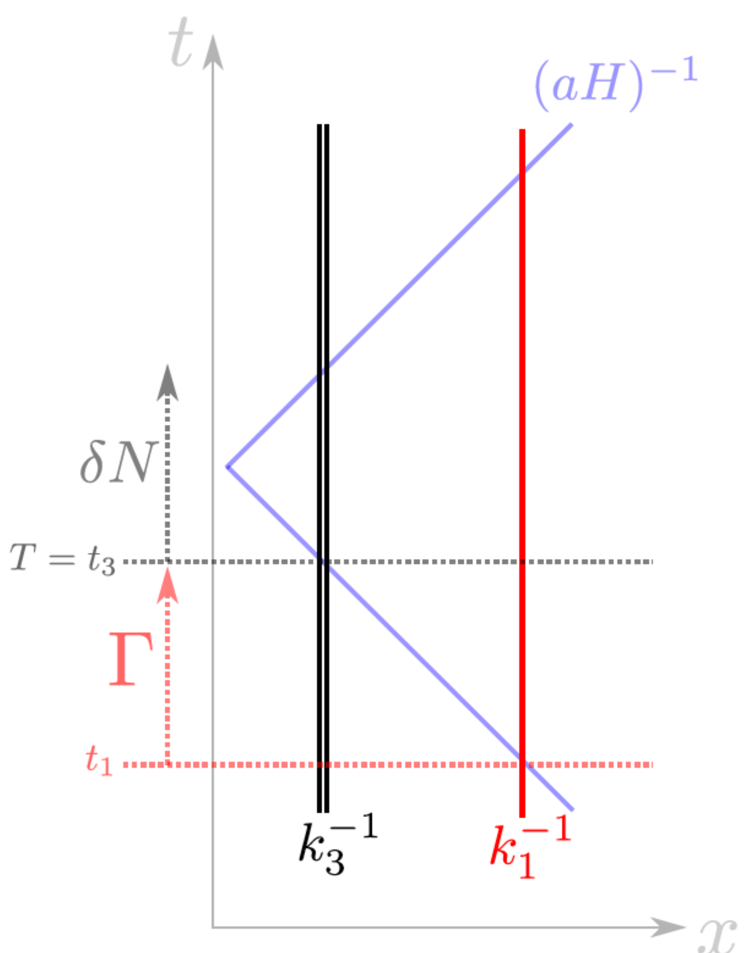


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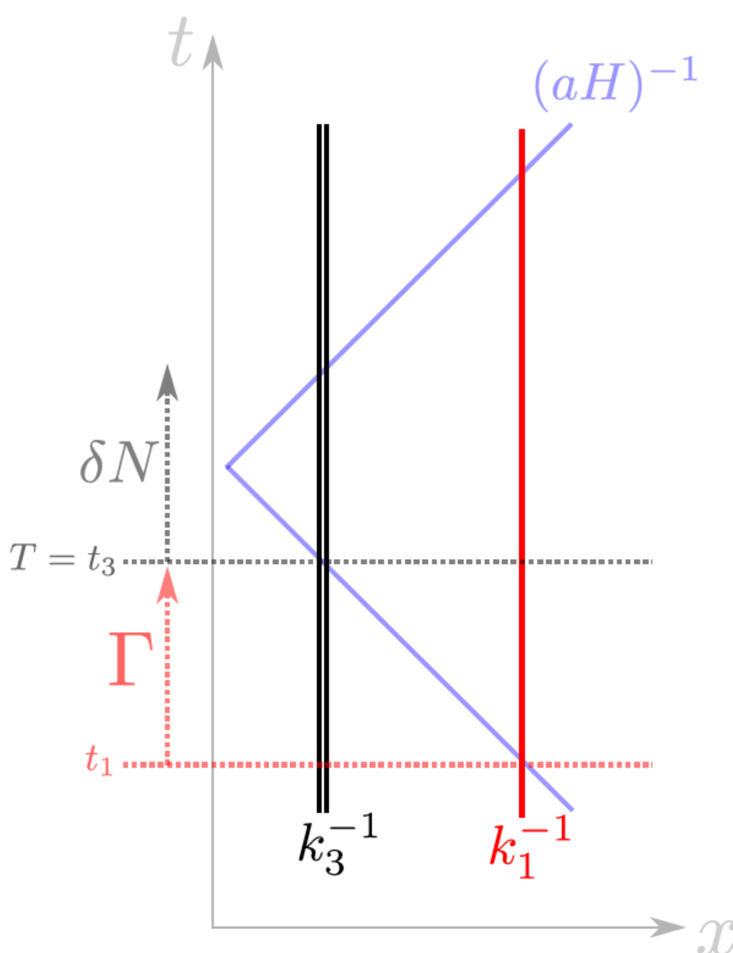


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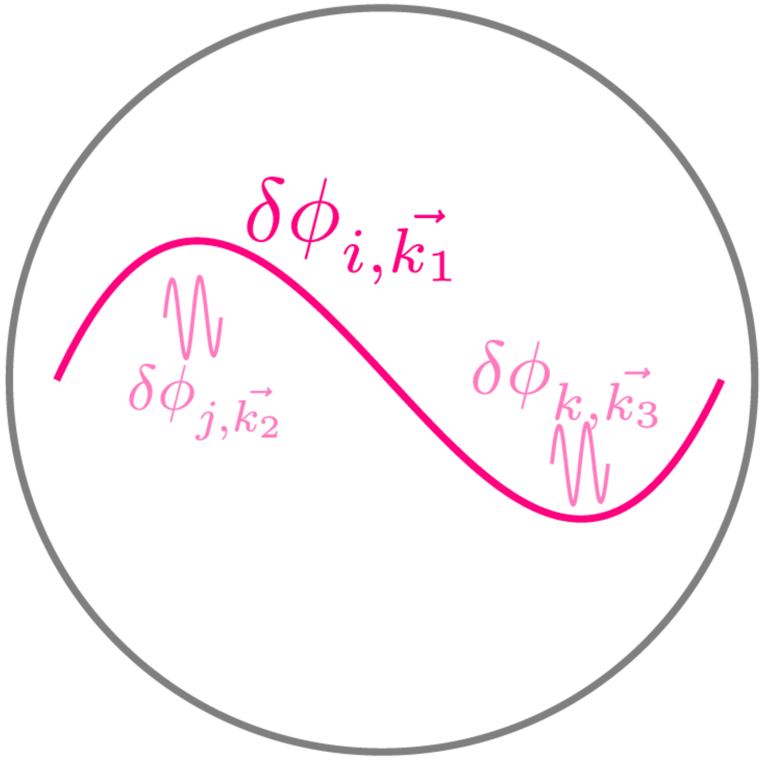

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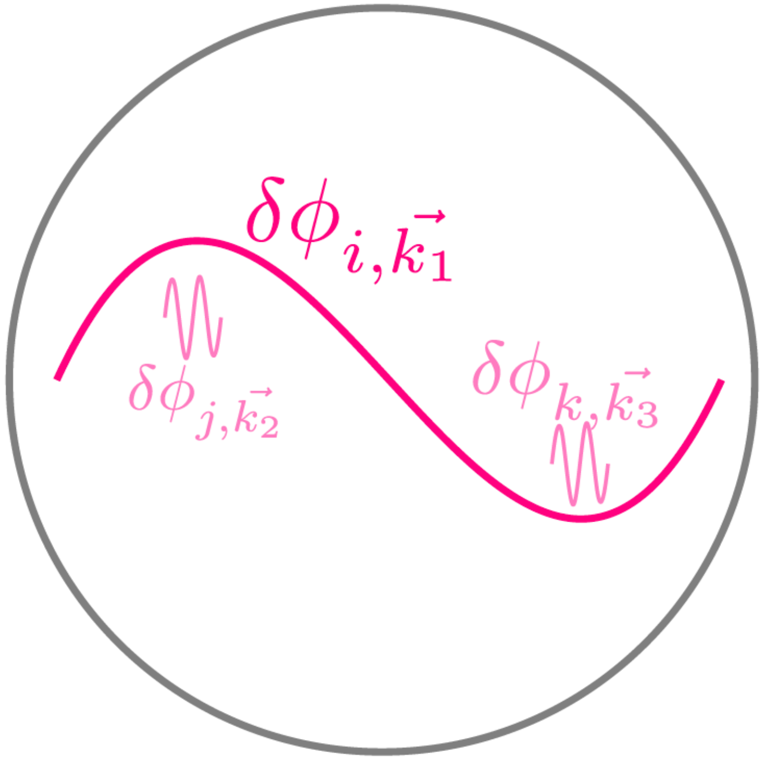


$$\lim_{k_1 \ll k_3, k_2} \alpha^{(3)}_{ijk}(k_1, k_2, k_3) \approx \Sigma^{(3)}_{im}(k_1) \frac{\partial}{\partial \phi^{(3)}_m} \Sigma^{(3)}_{jk}(k_3)$$

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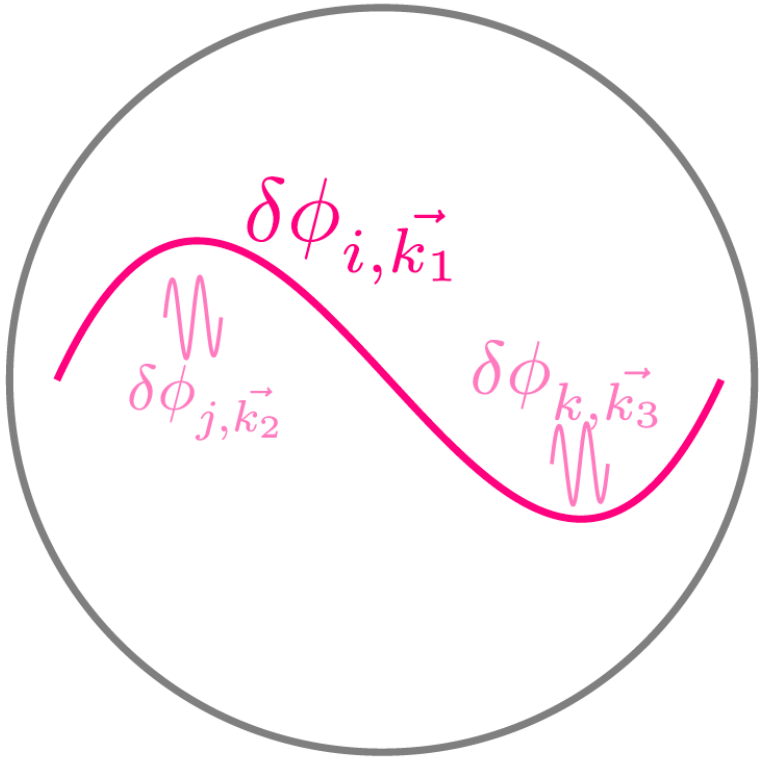


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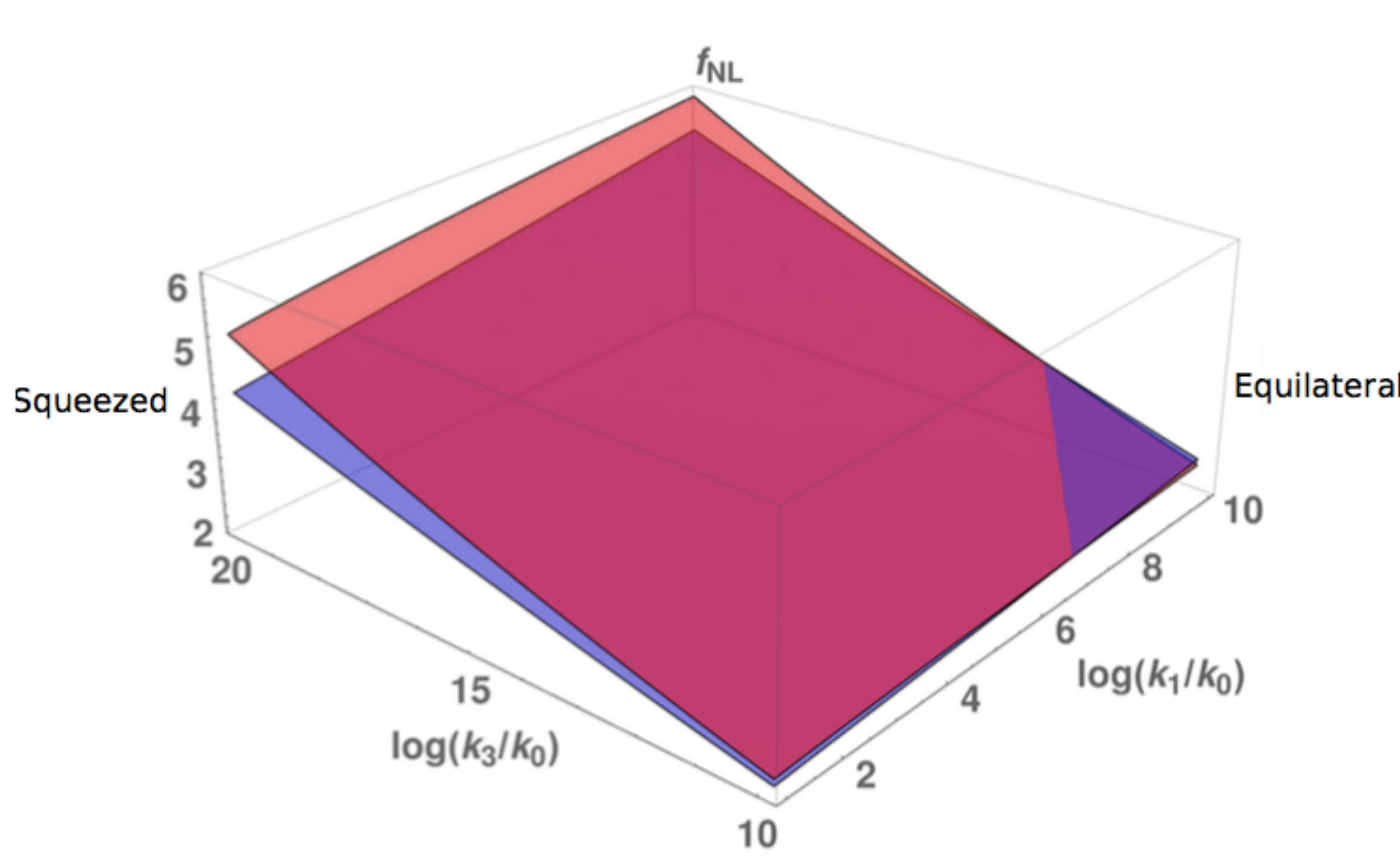
- Note similarity to Maldacena relation for curvature perturbation (c.f. previous results: Allen et al. '05; Li and Wang '08)

# Example: interacting curvaton

- Bispectrum in squeezed limit (relevant for e.g. Halo Bias)

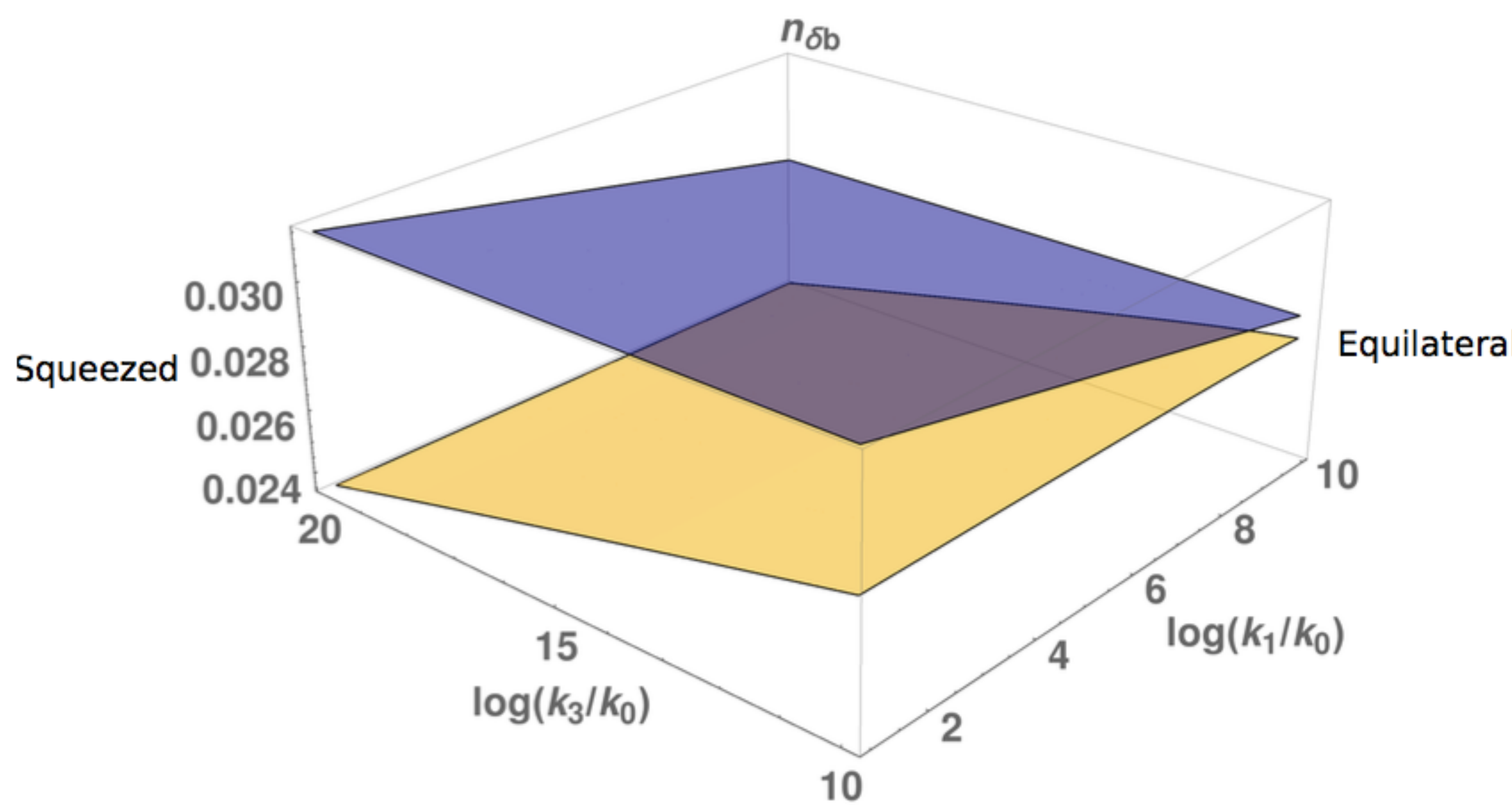
$$f_{NL}(k_1, k_2, k_3) \sim \frac{B_\zeta(k_1,k_2,k_3)}{P_\zeta(k_1)P_\zeta(k_3)}$$



# Example: interacting curvaton

- Spectral index of the Halo Bias

$$n_{\delta b} \sim \frac{d\log f_{\rm NL}(k_1,k_2,k_3)}{d\log k_1}$$



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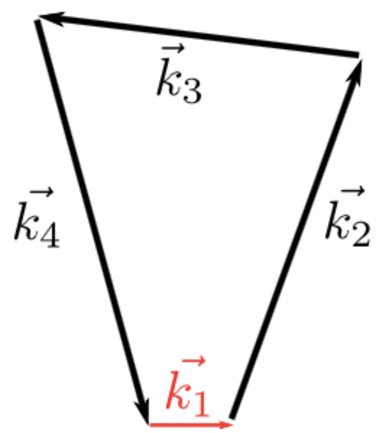

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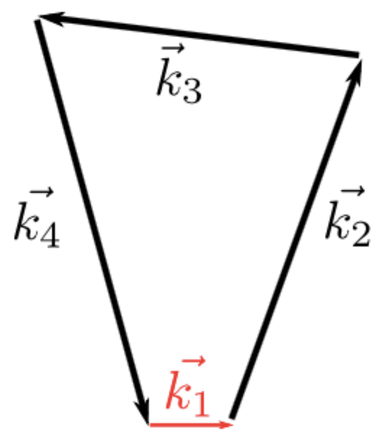


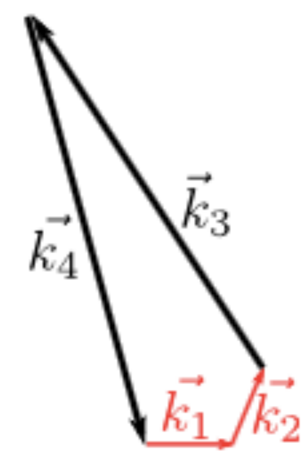

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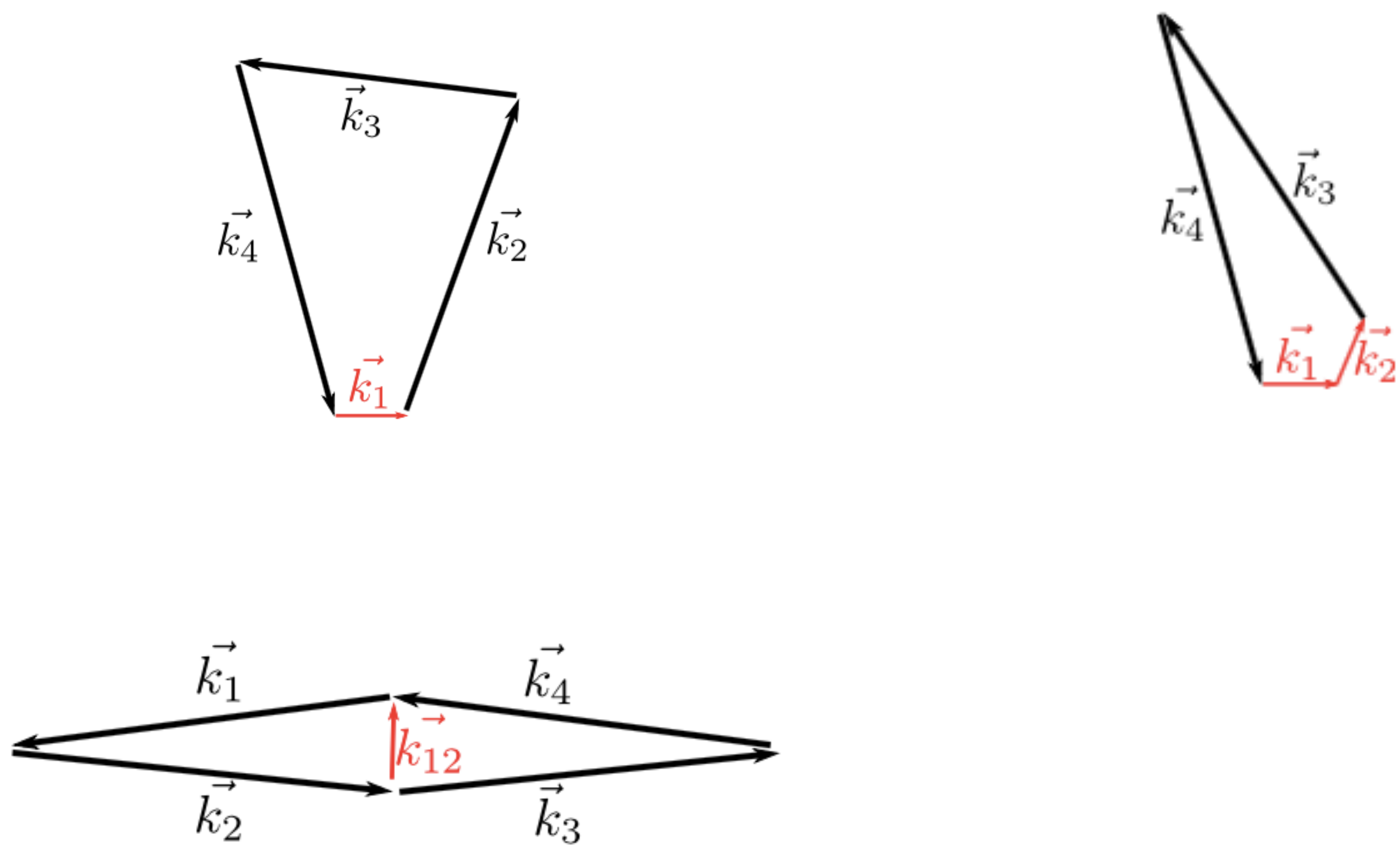

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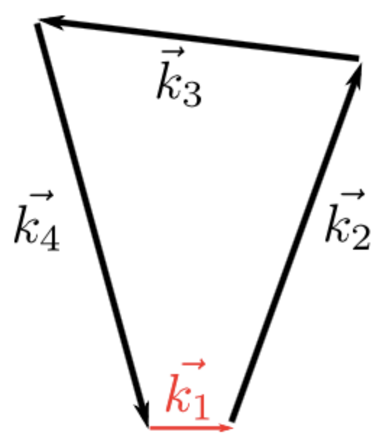


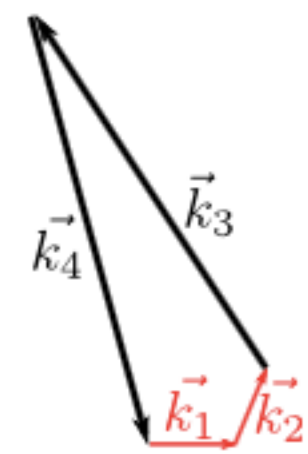


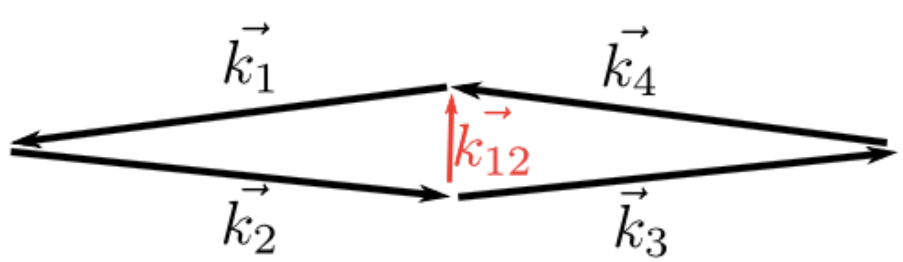


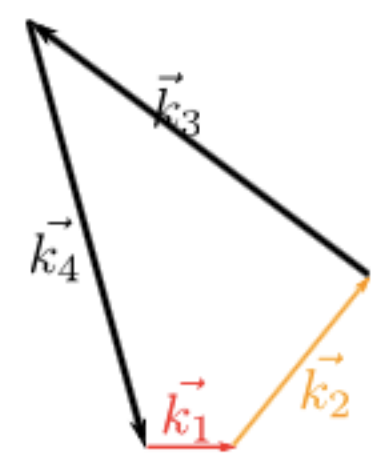

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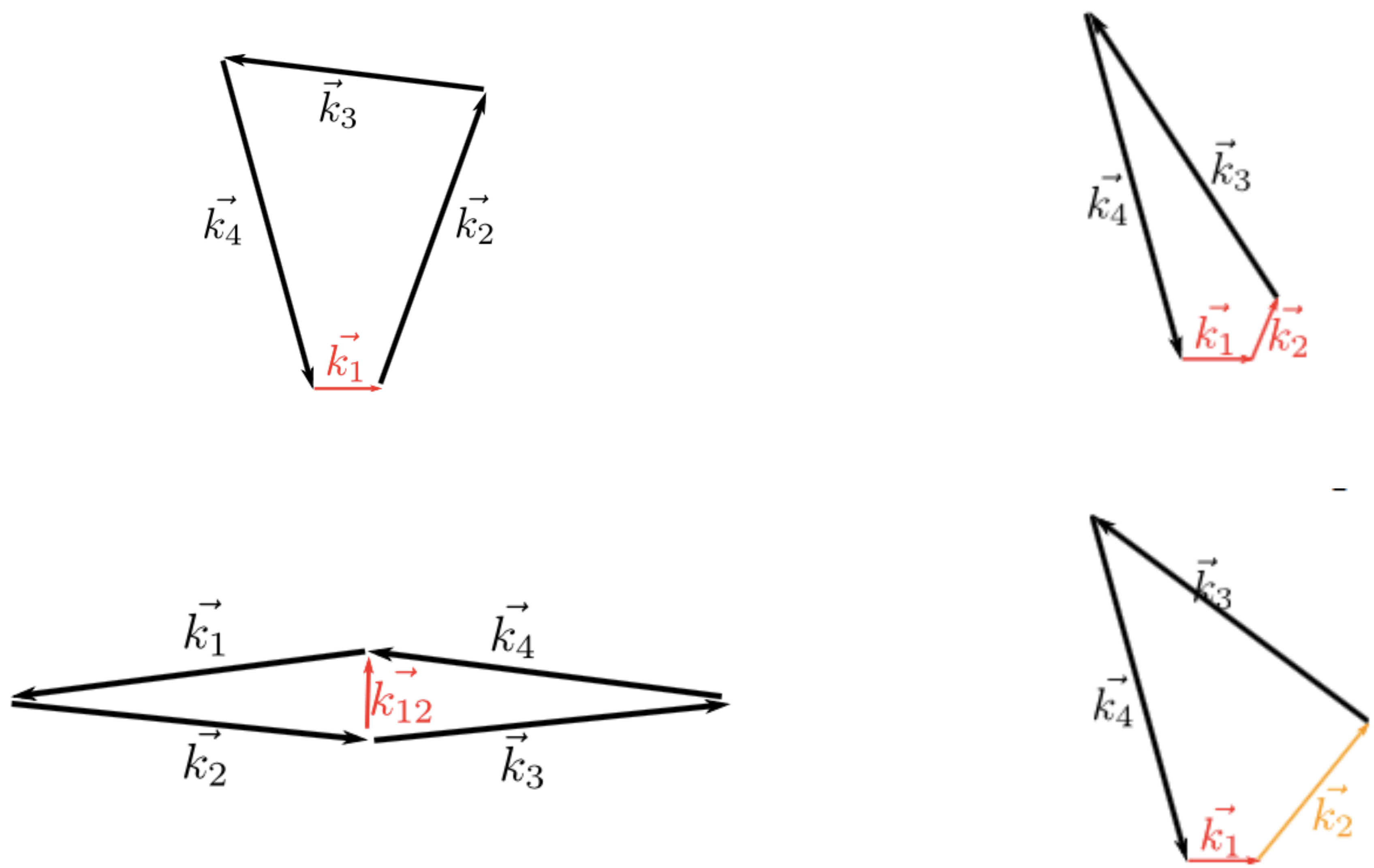


- Important to be able to calculate to compare with observation, and provides new insight to Suyama-Yamaguchi relation

# Conclusion

- Soft limits lead to interesting consistency relations
- We also simply need to be able to calculate correlations away from near equilateral configurations (to compare models with observations)
- We have presented an explicit discussion for the multiple field bispectrum in arXiv:1507.08629
- In ongoing work we are extending to the trispectrum